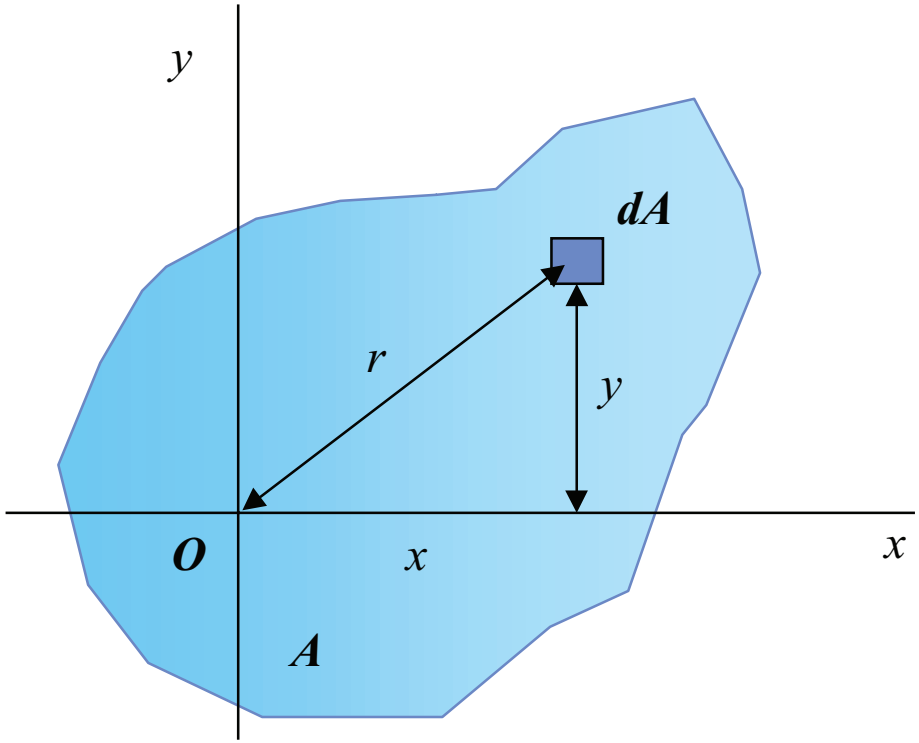
9.6 Polar Moment of Inertia



The ***polar moment of inertia of an area A*** with respect to the pole *O* is defined as

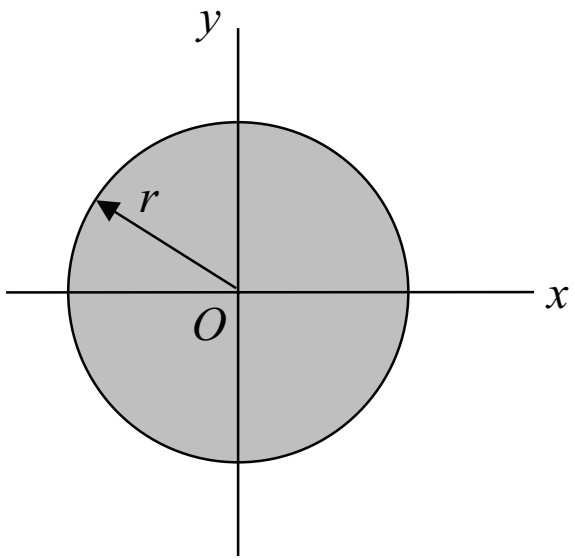
$$J_O = \int r^2 \, dA$$

The distance from O to the element of area dA is r. Observing that $r^2 = x^2 + y^2$, we established the relation

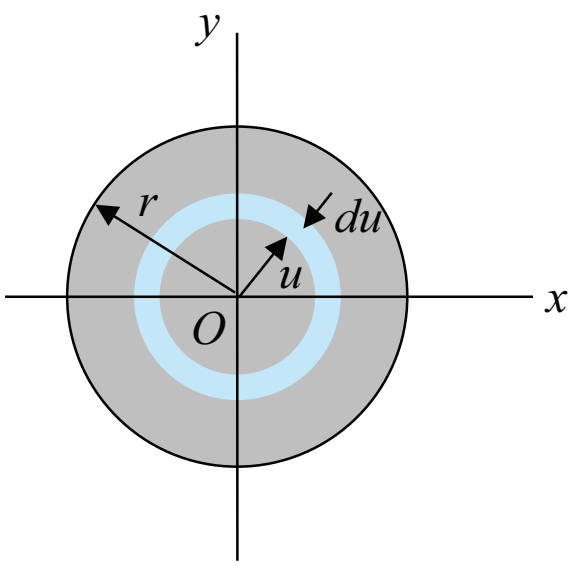
$$J_O = I_x + I_y$$

Example 9.9

(*a*) Determine the centroidal polar moment of inertia of a circular area by direct integration. (*b*) Using the result of part *a*, determine the moment of inertia of a circular area with respect to a diameter.



SOLUTION



a. Polar Moment of Inertia.

$$dJ_O = u^2\, dA \qquad dA = 2\pi u\, du$$

$$J_O = \int dJ_O = \int_0^r u^2 (2\pi u\, du) = 2\pi \int_0^r u^3 du$$

$$J_O = \frac{\pi}{2} r^4 \quad \Leftarrow$$

b. Moment of Inertia with Respect to a Diameter.

$$J_O = I_x + I_y = 2I_x$$

$$\frac{\pi}{2} r^4 = 2I_x$$

$$I_{diameter} = I_x = \frac{\pi}{4} r^4 \quad \Leftarrow$$