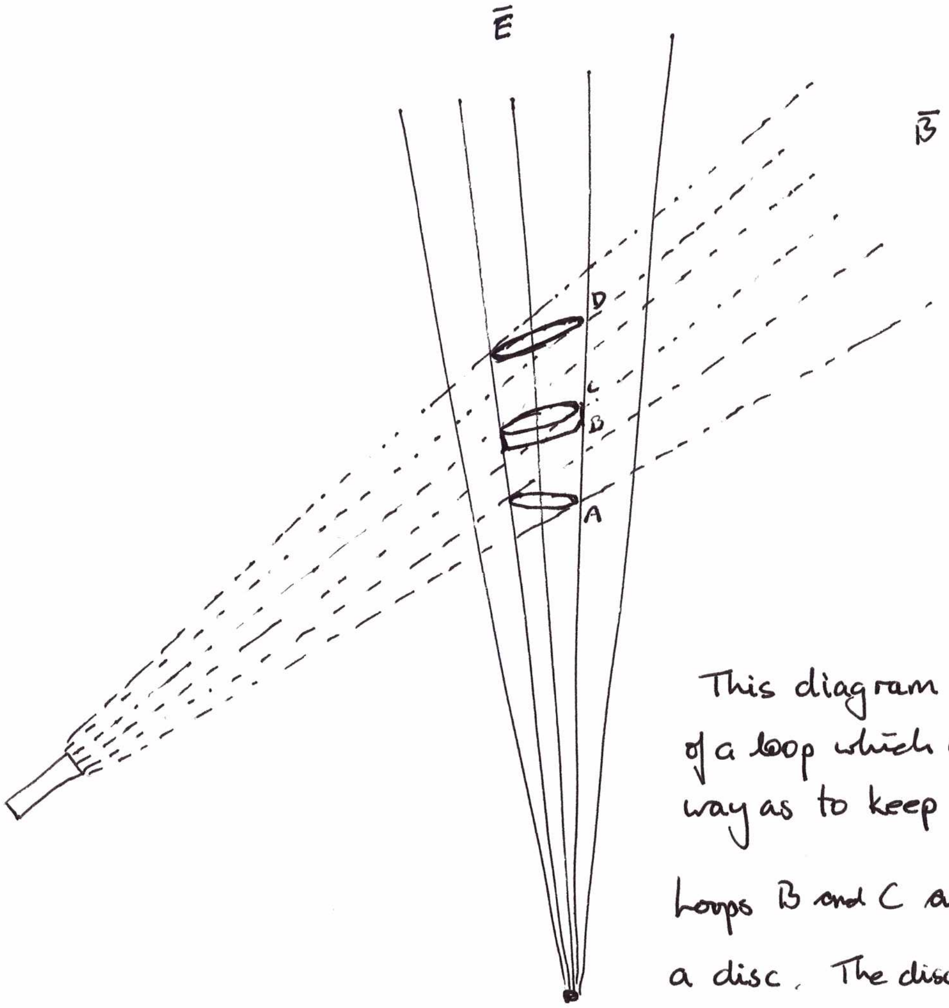

This diagram demonstrated the idea of a loop which can be moved in such a way as to keep both Φ_F and Φ_{*F} constant. Loops B and C are close together, creating a disc. The disc is made up of two flat surfaces connected by a thin surface which lies in the calumoid, made up from a collection of elementary bivectors $\Sigma = \frac{\partial P}{\partial a} \wedge \frac{\partial P}{\partial b}$. From the definition of the calumoid, it is clear that, as Φ_F and Φ_{*F} are equal on both faces of the disc, then

$\int \langle F, \Sigma \rangle \, da\, db$ and $\int \langle *F, \Sigma \rangle \, da\, db$ must be zero.

This does not, however, mean that $\langle F, \Sigma \rangle = 0$ and $\langle *F, \Sigma \rangle = 0$. In my diagram above, I have drawn it so that $\langle F, \Sigma \rangle = 0$ as the loop slides up and down lines of force so everywhere $*F$ is parallel to the surface of P - no bongs of the bell. The same cannot be said for ~~$*F$~~ F. Indeed it is possible to reverse the situation so that ~~$*F$~~ F is parallel to the surface, or even construct a calumoid where neither F nor $*F$ is parallel to the surface of the elementary bivectors.

Moving on to part (c):

$$\langle F, \Sigma \rangle = 0 \qquad \langle *F, \Sigma \rangle = 0.$$

$$\therefore \quad F_{\mu\nu}\left(\frac{\partial x^\mu}{\partial a}\frac{\partial x^\nu}{\partial b} - \frac{\partial x^\nu}{\partial a}\frac{\partial x^\mu}{\partial b}\right) da \wedge db = 0$$

and

$$*F_{\mu\nu}\left(\frac{\partial x^\mu}{\partial a}\frac{\partial x^\nu}{\partial b} - \frac{\partial x^\nu}{\partial a}\frac{\partial x^\mu}{\partial b}\right) da \wedge db = 0.$$

$$\Rightarrow \quad \frac{\partial x^\mu}{\partial a}\frac{\partial x^\nu}{\partial b} - \frac{\partial x^\nu}{\partial a}\frac{\partial x^\mu}{\partial b} = 0.$$

$$\therefore \quad \left.\begin{aligned} x^\mu &= x^\mu(\phi^\mu(a)) \\ x^\nu &= x^\nu(\phi^\nu(a)) \end{aligned}\right\} A$$

$$\text{or} \quad \left.\begin{aligned} x^\mu &= x^\mu(\theta^\mu(b)) \\ x^\nu &= x^\nu(\theta^\nu(b)) \end{aligned}\right\} B$$

But because μ, ν occurs in all combinations of $\mu\,(0 \ldots 3)$, $\nu\,(0 \ldots 3)$ there can be no mixing of the solutions, it is either ~~A~~ A or B. But a solution which depends on (a) only is a line not a surface. In part (d) there is a solution involving the two parameters a and b because $F = -E_x\, dt \wedge dx$ leads to $*F = -E_x\, dy \wedge dz$ so there are two sets of mutually exclusive μ and ν in $(0,1)$ and $(2,3)$. Combining A and B produces the answer given.