MyWork:

$2y'' + y' - y = x + 1;\ y(0) = 1,\ y'(0) = 0$

$2m^2 + m - 1 = 0 \Rightarrow m = \dfrac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} \Rightarrow m = -\dfrac{1}{4} \pm \dfrac{3}{4} \Rightarrow m = \dfrac{1}{2}, -1$

$\Rightarrow y_c = c_1 e^{-x} + c_2 e^{\frac{x}{2}}$

$y_p = v_1 e^{-x} + v_2 e^{\frac{x}{2}}$

$$W = det\left(\begin{bmatrix} e^{-x} & e^{\frac{x}{2}} \\ -e^{-x} & \frac{1}{2}e^{\frac{x}{2}} \end{bmatrix}\right) = \frac{1}{2}e^{-\frac{x}{2}} + e^{-\frac{x}{2}} = \frac{3}{2}e^{-\frac{x}{2}}$$

$$W_1 = det\left(\begin{bmatrix} 0 & e^{\frac{x}{2}} \\ x+1 & \frac{1}{2}e^{\frac{x}{2}} \end{bmatrix}\right) \Rightarrow W_1 = -(x+1)e^{\frac{x}{2}}$$

$$W_2 = det\left(\begin{bmatrix} e^{-x} & 0 \\ -e^{-x} & x+1 \end{bmatrix}\right) = (x+1)e^{-x}$$

$$\frac{W_1}{W} = \frac{-(x+1)e^{\frac{x}{2}}}{\frac{3}{2}e^{-\frac{x}{2}}} = -\frac{2}{3}(x+1)e^{x}$$

$$\frac{W_2}{W} = \frac{(x+1)e^{-x}}{\frac{3}{2}e^{-\frac{x}{2}}} = \frac{2}{3}(x+1)e^{-\frac{x}{2}}$$

$$v_1 = \int\left(-\frac{2}{3}(x+1)e^{x}\right)dx$$

$f_1 = -\dfrac{2}{3}(x+1),\ df_1 = -\dfrac{2}{3}dx,\ dg_1 = e^x dx,\ g_1 = e^x$

$$v_1 = -\frac{2}{3}(x+1)e^{x} + \int\frac{2}{3}e^{x}dx$$

$$v_1 = -\frac{2}{3}(x+1)e^{x} + \frac{2}{3}e^{x}$$

$$v_1 = -\frac{2}{3}x\,e^{x}$$

$$v_2 = \int \left(\frac{2}{3}(x+1)e^{-\frac{x}{2}} \right) dx$$

$$f_2 = \frac{2}{3}(x+1),\ df_2 = \frac{2}{3}dx,\ dg_2 = e^{-\frac{x}{2}}dx,\ g_2 = -2e^{-\frac{x}{2}}$$

$$v_2 = -\frac{4}{3}(x+1)e^{-\frac{x}{2}} + \int \frac{4}{3}e^{-\frac{x}{2}}dx$$

$$v_2 = -\frac{4}{3}(x+1)e^{-\frac{x}{2}} - \frac{8}{3}e^{-\frac{x}{2}}dx$$

$$v_2 = -\frac{4}{3}xe^{-\frac{x}{2}} - 4e^{-\frac{x}{2}}$$

$$y_p = \left(-\frac{2}{3}xe^{x} \right)e^{-x} + \left(-\frac{4}{3}xe^{-\frac{x}{2}} - 4e^{-\frac{x}{2}} \right)e^{\frac{x}{2}}$$

$$\Rightarrow y_p = -\frac{2}{3}x - \frac{4}{3}x - 4 \Rightarrow y_p = -2\mathrm{x} - 4$$