(take 2)

7.) $y'' - y = \cosh(x)$ $\qquad \cosh x = \dfrac{e^{x} + e^{-x}}{2}$

$m^2 - 1 = 0$

$m_1 = 1 \qquad m_2 = -1$

$y_c = c_1 e^{x} + c_2 e^{-x}$

$y_1 = e^{x} \; ; \; y_2 = e^{-x}$

$y_1' = e^{x} \; ; \; y_2' = -e^{x}$

$$e^{x} u_1' + e^{-x} u_2' = 0$$
$$e^{x} u_1' - e^{-x} u_2' = \cosh(x)$$

$$W = \begin{vmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{vmatrix} = -1 - 1 = \underline{\underline{-2}}$$

$$u_1' = \frac{\begin{vmatrix} 0 & e^{-x} \\ \cosh(x) & -e^{-x} \end{vmatrix}}{-2}$$

$$= -\frac{e^{-x}\cosh(x)}{-2} = \frac{e^{-x}}{2}\left(\frac{e^{x} + e^{-x}}{2}\right)$$

$$= \left(\frac{1}{4} + \frac{e^{-2x}}{4}\right)$$

$$u_2' = \frac{\begin{vmatrix} e^x & 0 \\ e^x & \cosh(x) \end{vmatrix}}{-2} = \frac{-e^x \cosh(x)}{2}$$

$$= -\frac{e^x}{2}\frac{(e^x + e^{-x})}{2} = \left(-\frac{e^{2x}}{4} - \frac{1}{4}\right)$$

$$u_1 = \int \left(\frac{1}{4} + \frac{e^{-2x}}{4}\right) dx \qquad \begin{aligned} u &= -2x \\ du &= -2\,dx \\ dx &= -\tfrac{1}{2}\,du \end{aligned}$$

$$= \left(\frac{1}{4}x - \frac{e^{-2x}}{8}\right)$$

$$u_2 = \int \left(-\frac{e^{2x}}{4} - \frac{1}{4}\right) dx = \left(-\frac{e^{2x}}{8} - \frac{1}{4}x\right)$$

$$y = c_1 e^x + c_2 e^{-x} + \frac{1}{4} x e^x - \frac{e^x e^{-2x}}{8}$$

$$- \frac{e^{-x} e^{2x}}{8} - \frac{1 x e^{-x}}{4}$$

$$= c_1 e^x + c_2 e^{-x} + \frac{1}{4} x (e^x - e^{-x}) \; - \frac{1}{8}(e^x + e^{-x})$$

according to the answer in the back of the book this term is gone