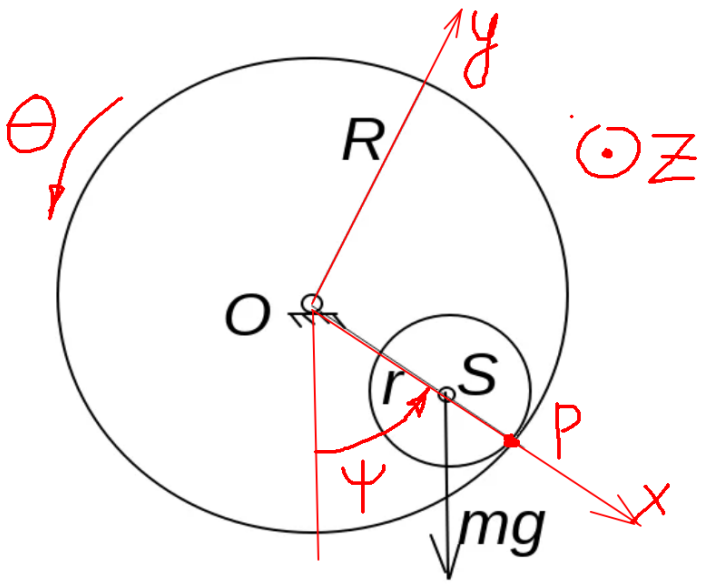


Figure 1:

θ — rotation angle of the cylinder; $\boldsymbol{v}_S = \dot{\psi}(R-r)\boldsymbol{e}_y$ is the velocity of the center of the disk; $\boldsymbol{\omega} = \omega \boldsymbol{e}_z$ is the angular velocity of the disk.

From the no-slip assumption by the standard kinematic relation

$$\boldsymbol{v}_S = \boldsymbol{v}_P + \boldsymbol{\omega} \times \boldsymbol{PS}, \quad \boldsymbol{PS} = -r\boldsymbol{e}_x, \quad \boldsymbol{v}_P = R\dot{\theta}\boldsymbol{e}_y$$

we find:

$$\omega = \rho\dot{\theta} - (\rho - 1)\dot{\psi}, \quad \rho = R/r.$$

Lagrangian:

$$L(\theta, \psi, \dot{\theta}, \dot{\psi}) = \frac{1}{2}J\dot{\theta}^2 + \frac{m}{2}\dot{\psi}^2(R-r)^2 + \frac{1}{2}I\omega^2 + mg(R-r)\cos\psi.$$

First integral associated with the cyclic coordinate θ:

$$\frac{\partial L}{\partial \dot{\theta}} = J\dot{\theta} + I\rho(\rho\dot{\theta} - (\rho-1)\dot{\psi}) = c_1. \tag{1}$$

For motions starting from rest, $c_1 = 0$. Integrating equation (1), we obtain

$$J\theta + I\rho(\rho\theta - (\rho-1)\psi) = c_2.$$

This is the key observation; the remainder is standard. The proof can be completed by considering the Routhian, which turns out to be the Lagrangian of a mathematical pendulum.