SECTION IV.

Determination of the motion of a fluid about a sphere which moves uniformly with a small velocity. Justification of the application of the solutions obtained in Sections II. and III. to cases in which the extent of oscillation is not small in comparison with the radius of the sphere or cylinder. Discussion of a difficulty which presents itself with reference to the uniform motion of a cylinder in a fluid.

39. Let a sphere move in a fluid with a uniform velocity V, its centre moving in a right line; and let the rest of the notation be the same as in Section II. Conceive a velocity equal and opposite to that of the sphere impressed both on the sphere and on the fluid, which will not affect the relative motion of the sphere and fluid, and will reduce the determination of the motion of the fluid to a problem of steady motion. Then we have for the equations of condition

$$R = 0, \qquad \Theta = 0, \text{ when } r = a \ldots\ldots\ldots\ldots\ldots (116);$$
$$R = -V\cos\theta,\ \Theta = V\sin\theta, \text{ when } r = \infty \ \ldots\ldots (117).$$

The equations of condition, as well as the equations of motion, may be satisfied by supposing ψ to have the form $\sin^2\theta f(r)$. We get from (20′), by the same process as that by which (33), (34) were obtained,

$$\left(\frac{d^2}{dr^2} - \frac{2}{r^2}\right)^2 f(r) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (118),$$

the only difference being that in the present case the equation (20′) cannot be replaced by the two (22), (23), which become identical, inasmuch as the velocity of the fluid is independent of the time.

The integral of (118) is

$$f(r) = Ar^{-1} + Br + Cr^2 + Dr^4 \ \ldots\ldots\ldots\ldots\ldots (119),$$

which gives

$$R = \frac{1}{r^2\sin\theta}\frac{d\psi}{d\theta} = 2\cos\theta\,(Ar^{-3} + Br^{-1} + C + Dr^2),$$

$$\Theta = -\frac{1}{r\sin\theta}\frac{d\psi}{dr} = \sin\theta\,(Ar^{-3} - Br^{-1} - 2C - 4Dr^2).$$