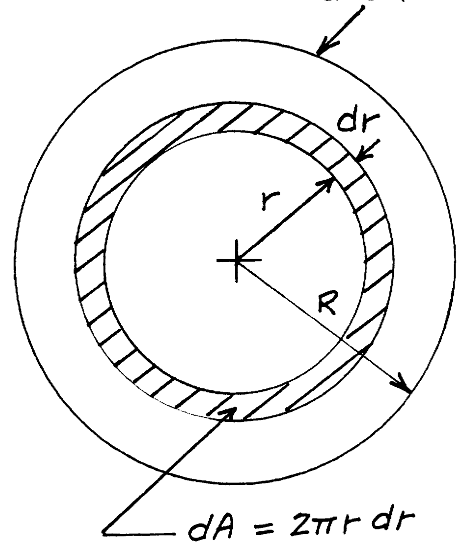
5.16

5.16 An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c \left(\frac{R-r}{R}\right)^{1/n} \hat{\mathbf{i}}$$

where u_c = centerline velocity, r = local radius, R = pipe radius, and $\hat{\mathbf{i}}$ = unit vector along pipe centerline. Determine the ratio of average velocity, $\bar{u}$, to centerline velocity u_c for **(a)** $n = 4$; **(b)** $n = 6$; **(c)** $n = 8$; **(d)** $n = 10$



For any cross section area

$$\dot{m} = \rho A \bar{u} = \int_A \rho \vec{V} \cdot \hat{n} \, dA$$

Also

$$\vec{V} \cdot \hat{n} = \vec{V} \cdot \hat{i} = u_c \left(\frac{R-r}{R}\right)^{\frac{1}{n}}$$

Thus for a uniformly distributed density, ρ, over area A

$$\bar{u} = \frac{\int_0^R u_c \left(\frac{R-r}{R}\right)^{\frac{1}{n}} 2\pi r \, dr}{\pi R^2}$$

and

$$\frac{\bar{u}}{u_c} = 2\int_0^R \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = \frac{2n^2}{2n^2 + 3n + 1}$$

n	$\frac{\bar{u}}{u_c}$
4	0.711
6	0.791
8	0.837
10	0.866