Ans 3, $\frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt}$

$\frac{ds}{dt} = f_x \frac{dx}{ds} + f_y \frac{dy}{ds} + f_z \frac{dz}{ds}$

a) $f(x,y,z) = xyz^{10}$

$x = t^3$

$y = \ln(s^2\sqrt{t}) = 2\ln(s) + \ln(\sqrt{t})$

$z = e^{st}$

$f(x) = yz^{10}$

$\frac{dx}{dt} = 3t^2$ $\quad f(x)_t = 3t^2 y z^{10}$

$\frac{dx}{ds} = 0$ $\quad f(x)_s = 0$

$f(x)\frac{dx}{dt} = 3t^2 \cdot (2\ln(s) + \ln(\sqrt{t})) \cdot e^{st}$

$f(x)\frac{ds}{dt} = 0$

$f(y) = xz^{10}$

$\frac{dy}{dt} = \frac{d}{dt}((2\ln(s)) + \frac{d}{dt}(\ln\sqrt{t}))$

$$\frac{d}{dt}(2\ln(s)) = 0$$

$$\frac{d}{dt}(\ln(\sqrt{t})) = \frac{1}{2t}$$

$$= 0 + \frac{1}{2t} = \frac{1}{2t}$$

$$f(y)_t = x\left(\frac{1}{2t}\right) z^{10}$$

$$f(y)_s = x\left(\frac{2}{s}\right) z^{10}$$

$$\frac{dy}{ds} = \frac{d}{ds}(2\ln(s)) + \frac{d}{ds}(\ln(\sqrt{t}))$$

$$\frac{d}{ds}(2\ln(s)) = \frac{2}{s}$$

$$\frac{d}{ds}((\ln\sqrt{t})) = 0$$

$$= \frac{2}{s} + 0 = \frac{2}{s}$$

$$f(y)\frac{dy}{dt} = t^{\cancel{3}}\left(\frac{1}{2\cancel{t}}\right) z^{10}$$

$$= \frac{t e^{10st}}{2}$$

$$f(y)\frac{dy}{ds} = t^3\left(\frac{2}{s}\right) e^{10st}$$

$$= \frac{2t^3 e^{10st}}{s}$$

$f(z) = 10xy\hat{z}^9$

$f(z)_t = 10xy(se^{st})^9$

$f(z)_s = 10xy(te^{st})^9$

$\frac{dz}{dt} = se^{st}$

$\frac{dz}{ds} = te^{st}$

$f(z)\frac{dz}{dt} = 10xy(se^{st})^9$

$= 10t^3 \cdot (2\ln(s) + \ln\sqrt{t}) \cdot (se^{st})^9$

$f(z)\frac{ds}{dt} = 10t^3(2\ln(s) + \ln(\sqrt{t}))(te^{st})^9$

$\frac{\partial f}{\partial t} = 3t^2 \cdot (2\ln(s) + \ln(\sqrt{t})) \cdot e^{st} + \frac{te^{st}}{2} + 10t^3$

$(2\ln(s) + \ln\sqrt{t}) \cdot (se^{st})^9$

$= 3t^2 \ln(s^2\sqrt{t})\, e^{\boxed{st}} + \frac{te^{\boxed{st}}}{2} + 10t^3 \ln(s^2\sqrt{t})$

$(\boxed{s}e^{st})^{\boxed{9}}$

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$$\frac{\partial f}{\partial s} = 0 + \frac{2te^{10st}}{s} + 10t^3(2\ln(s) + \ln(\sqrt{t}))(te^{st})^9$$

$$= \frac{2te^{10st}}{s} + 10t^3(2\ln(s) + \ln(\sqrt{t}))(te^{st})^9$$

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b) $f = x^{10} + y^{11} + z,$

$x = \sqrt{t+s^2}$

$y = \frac{t}{s}$

$z = s.t$

$$f(x) = 10x^9 + \boxed{\frac{d}{dx}(z)} \to 0$$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t+s^2}} \frac{d}{dt}(t+s^2) \to 1$$

$$= \frac{1}{2\sqrt{t+s^2}}$$

$$\frac{dx}{ds} = \frac{1}{2\sqrt{t+s^2}} \frac{d}{ds}(t+s^2)$$

$$\boxed{\frac{d}{ds}(t+s^2) = 2s}$$

$$= \frac{s}{\sqrt{t+s^2}}$$

$$\boxed{\begin{aligned} f(x)_t &= 10\left(\frac{1}{2\sqrt{t+s^2}}\right)^9 \\ f(x)_s &= 10\left(\frac{s}{\sqrt{t+s^2}}\right)^9 \end{aligned}}$$

$$\cancel{f(h)\,\frac{dh}{dt}/2}$$

$$f(x)\frac{dx}{dt} = 10\left(\frac{\cancel{1}}{2\sqrt{t+s}}\right)$$

$$= 10 \cdot \frac{1}{2^9 (t+s^2)^{9/2}} = \frac{\cancel{2}\cdot 5}{\cancel{2}^9 (t+s^2)^{9/2}}$$

$$= \frac{5}{2^8 (t+s^2)^{9/2}}$$

$$= \frac{5}{256\,(t+s^2)^{9/2}}$$

$$f(x)\frac{dx}{ds} = 10\left(\frac{s}{\sqrt{t+s^2}}\right)^9$$

$$= 10\left(\frac{s}{\sqrt{t+s^2}}\right)^9$$

$$= \frac{10 \cdot s^9}{(s^2+t)^{9/2}} = \frac{s^9 \, 10}{(t+s^2)^{9/2}}$$

$f(y) = 11y^{10} + \cancel{0}$

$$\frac{dy}{dt} = \frac{1}{s} \qquad \Bigg| \qquad t\frac{d}{ds}(s^{-1})$$

$$\frac{dy}{ds} = -\frac{t}{s^2} \qquad \Bigg| \qquad = t(-1 . s^{-1-1})$$

$$f(x)\frac{dx}{dt} = 11\left(\frac{1}{s}\right)^{10}$$

$$f(x)\frac{dx}{ds} = -11\left(\frac{-t}{s^2}\right)^{10}$$

$$f(z) = 1$$

$$\frac{dz}{dt} = s \qquad \frac{dz}{ds} = t$$

$$f(z)\frac{dx}{dt} = 1$$

$$f(z)\frac{dx}{ds} = 1$$

$$\frac{\partial f}{\partial t} = \frac{5}{\boxed{256}\,(t+s^2)^{\boxed{9/2}}} + 11\left(\frac{1}{s}\right)^{\boxed{10}} + \boxed{1}$$

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$$\frac{\partial f}{\partial s} = \frac{s^{\boxed{9}} \cdot 10}{(t+s^2)^{\boxed{9/2}}} + 11\left(\frac{-t}{s^2}\right)^{\boxed{10}} + \boxed{1}$$

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> ***restart Q3***

> $w := x \cdot y \cdot z^{10}$

$$w := x \cdot y \cdot z^{10} \tag{1}$$

> $x := t^3$

$$x := t^3 \tag{2}$$

> $y := \ln\left(s^2 \cdot \mathrm{sqrt}(t)\right)$

$$y := \ln\left(s^2 \cdot \sqrt{t}\right) \tag{3}$$

> $z := \mathrm{e}^{t \cdot s}$

$$z := \mathrm{e}^{t \cdot s} \tag{4}$$

> $m := \mathit{diff}(w, t)$

$$m := 3\, t^2 \cdot \ln\left(s^2 \cdot \sqrt{t}\right) \cdot \left(\mathrm{e}^{t \cdot s}\right)^{10} + \frac{t^3 \cdot \left(\frac{s^2 \cdot \frac{1}{\sqrt{t}}}{s^2 \cdot \sqrt{t}}\right) \cdot \left(\mathrm{e}^{t \cdot s}\right)^{10}}{2} + 10\, t^3 \cdot \ln\left(s^2 \cdot \sqrt{t}\right) \cdot \left(\left(\mathrm{e}^{t \cdot s}\right)^{10} s\right) \tag{5}$$

> $m := \mathit{diff}(w, s)$

$$m := 2\, t^3 \cdot \left(\frac{s \cdot \sqrt{t}}{s^2 \cdot \sqrt{t}}\right) \cdot \left(\mathrm{e}^{t \cdot s}\right)^{10} + 10\, t^3 \cdot \ln\left(s^2 \cdot \sqrt{t}\right) \cdot \left(\left(\mathrm{e}^{t \cdot s}\right)^{10} t\right) \tag{6}$$

> *restart*

> $r := x\text{^}10 + y\text{^}11 + z$

$$r := y^{11} + x^{10} + z \tag{7}$$

> $x := \mathrm{sqrt}\left(t + s^2\right)$

$$x := \sqrt{s^2 + t} \tag{8}$$

> $y := \frac{t}{s}$

$$y := \frac{t}{s} \tag{9}$$

> $z := s \cdot t$

$$z := s \cdot t \tag{10}$$

> $\mathit{diff}(r, t)$

$$\frac{11\, t^{10}}{s^{11}} + 5\left(s^2 + t\right)^4 + s \tag{11}$$

> $\mathit{diff}(r, s)$

$$-\frac{11\, t^{11}}{s^{12}} + 10\left(s^2 + t\right)^4 s + t \tag{12}$$

> ***restart Q5***

> $\mathit{equation1} := y = x\text{^}3 + 2.\, x\text{^}2 + 7.\, x$

$$\mathit{equation1} := y = x^3 + 2.\, x^2 + 7.\, x \tag{13}$$

> $implicitdiff(equation1, x, y)$

$$\frac{1}{3.\,x^2 + 4.\,x + 7.} \tag{14}$$

> $equation2 := x^3 \cdot y^{10} - x \cdot y^3 = 10.\,x$

$$equation2 := x^3 \cdot y^{10} - x \cdot y^3 = 10.\,x \tag{15}$$

> $implicitdiff(equation2, x, y)$

$$RootOf\left(-3\,\left(x^2\,\_Z\right) \cdot y^{10} - 10\,x^3 \cdot y^9 + \_Z \cdot y^3 + 3\,x \cdot y^2 + 10\,\_Z\right) \tag{16}$$

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