Patterns in integrals

a. Use a computer algebra system to evaluate the following integrals. (I used MAPLE)

$$\int x \cdot e^x \, dx = (x-1)\, e^x$$

$$\int x^2 \cdot e^x \, dx = \left(2 - 2x + x^2\right) e^x$$

$$\int x^3 \cdot e^x \, dx = \left(-6 + 6x - 3x^2 + x^3\right) e^x$$

$$\int x^4 \cdot e^x \, dx = \left(24 - 24x + 12x^2 - 4x^3 + x^4\right) e^x$$

$$\int x^5 \cdot e^x \, dx = \left(-120 + 120x - 60x^2 + 20x^3 - 5x^4 + x^5\right) e^x$$

b. Based on your pattern in part a, guess the value of $\int x^6 e^x \, dx$. Then use your CAS to check your guess.

This was my guess: $e^x(x^6 - 6x^5 + 30x^4 - 120x^3 + 360x^2 - 720x + 720)$

When I check my CAS I got:

$\int x^6 \cdot e^x \, dx = \left(720 - 720x + 360x^2 - 120x^3 + 30x^4 - 6x^5 + x^6\right) e^x$ which I guessed correctly. The pattern was simple to recognize and guessing the answer was not a problem.

c. Based on the patterns in a and b, make a conjecture as to the value of the integral $\int x^n e^x \, dx$ when n is a positive integer. Now this is my problem, this is what I came up with and where im stuck because I know its not correct.

$$\sum_{i=0}^{n} \frac{n!}{i!} x^i e^x$$

d. Use a mathematical induction to prove the conjecture you made in part c.