

Conical Springs

AUGUST RODENSCHATZ

The use of conical springs is not a common practice. This is probably due to lack of engineering information and the difficulties in adjusting machinery for winding them. There are occasions when conical springs best fit into a design. These conditions exist when axial space is limited and radial space is available.

The writer has come across a deflection formula which can be used for extension springs. The same formula will hold for compression springs as long as no coil of the springs pots; for until this condition is reached, the gradient of the spring has remained constant.

In order to know the characteristics of the load deflection curve of a spring the pitch of successive coils must follow certain laws. We consider first the spring in which all coils seat simultaneously at some predetermined load, which of course exceeds the useful load. The well-known formula (1) expresses deflection per coil and can be used to express the elemental deflection of the conical coil.

$$\frac{F}{N} = \frac{8PD^3}{Gd^4} = \frac{dF}{dN} \quad (1)$$

- N_t = total number of turns
- F_t = deflection of entire spring
- L = free length
- P = load
- G = modulus of elasticity in shear
- $M = kGd^4$
- D = mean diameter
- d = wire diameter

We note that D , the mean diameter of the elemental section, varies for all sections, and must be given in terms of the variable N . Equation (2) expresses this relation in which N is the number of turns beginning from the left

$$D = (A + 2kN) \quad (2)$$

Equations (3) and (4) naturally follow by substituting (2) in (1) and integrating

$$dF = \frac{8P}{Gd^4} (A + 2kN)^3 dN$$

$$\int_0^{F_t} dF = \frac{4P}{kGd^4} \times \int_0^{N_t} (A + 2kN)^3 d(A + 2kN) \quad (3)$$

$$F_t = \frac{P}{M} \left[(A + 2kN)^4 \right]_0^{N_t} \quad (4)$$

Equation (4) gives the total deflection, P being taken as the load at which simultaneous seating takes place. For all other loads, deflections are proportional to the load. The deflection of any one coil, say (N) to ($N+1$), may be found by use of (4) between limits ($N+1$) and (N). If b is added to this deflection, we then have the pitch of that coil. If k is equal to or greater than d , b will be zero and the spring will be solid at length d . Equation (5) gives the free length of the spring.

$$L = F_t + bN_t + d \quad (5)$$

This formula may be modified to

common practice of having $1\frac{1}{4}$ solid coils at each end and ground. Then F_t should be for the limits ($N_t - 1\frac{1}{4}$) and ($1\frac{1}{4}$). For the ground ends, the d drops out entirely.

Another interesting problem is to design the spring so that seating occurs at the point in the wire where a predetermined stress is reached. By this we mean the spring begins seating at the large diameter progressively moving to the smaller end as the load increases. Here again we use the standard formula (6) from any handbook giving the load stress relation.

$$P = \frac{\pi S_{max} d^3}{8Y(A + 2kN)} \quad (6)$$

S = stress
 Y = Wahl stress factor

$$Q = \frac{\pi S_{max}}{6kYGd}$$

The Wahl factor has been included and an average value for the range of D/d ratio can be used in each instance. The value of P in (6) is substituted in (3) to form equation (7)

$$dF = \frac{\pi S_{max}}{Y G d} (A + 2kN)^3 dN$$

$$\int_0^{F_t} dF = \frac{\pi S_{max}}{2kYGd} \times \int_0^{N_t} (A + 2kN)^3 d(A + 2kN) \quad (7)$$

The free length of the spring may be found by using the value of (8) in equation (5). The deflection and pitch of any one coil are found in the same manner as explained above using values derived from equation (8).

$$F_t = Q \left[(A + 2kN)^4 \right]_0^{N_t} \quad (8)$$

The load deflection curve (P - F) may now be constructed as follows: Equation (6) is put into form (9) in which N_s is that point in the spring which divides it into the seated and unseated portions at load P .

$$(A + 2kN_s) = \frac{\pi S_{max} d^3}{8YP}$$

or
$$N_t = \frac{\pi S_{max} d^3}{16kYP} - \frac{A}{2k} \quad (9)$$

If N_s is greater than N_t , the deflection will follow formula (4) and will be a straight line. If N_s is less than N_t , the value of F (4) limits N_s and O must be added to F (8) limits N_t to N_s to get total deflection.

