

[b]1. The problem statement, all variables and given/known data[/b]

"Determine the shape of a bullet "front" so as to have minimum air resistance. Let the radius of the bullet be "r". Allow the distance along the axis of symmetry from where the radius is zero to where it becomes r to be 2r. Consider the bullet to be of the hollow point design. That is consider the bullet starts of flat for a small distance 'a', and then begins a curved path. In order to consider air resistance let the air molecules be at rest---Then consider all molecules contained inside a cylinder of radius r to be elastically scattered from a bullet that is moving with velocity v through the cylinder of air. The kinetic energy received by the air molecules that get displaced by the bullet must have come from the bullet. This approach will give the energy loss of the bullet in terms of the shape of the bullet. Calculate the rate of energy loss by this "best" shape as compared to the worst shaped bullet (ratio)."

This is our project question for a senior/graduate level math methods course.

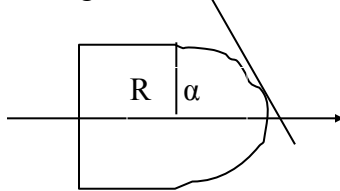
[b]2. Relevant equations[/b]

$$(\partial F / \partial y) - d/dx(\partial F / \partial y') = 0$$

[b]3. The attempt at a solution[/b]

We are trying to calculate the kinetic energy loss by collisions with air particles, which we assume to be completely elastic.

The bullet head, not including the shell casing, is of a length, 2R, which is twice the maximum radius. The contour of one side of the bullet could be defined as a function $f(x)=r$, and we take the volume of the bullet to be a solid of revolution around the x axis, with the x axis being the line of symmetry. The bullet tapers off at the nose to a small flat region of undefined slope which we've dubbed length "a."



(crude drawing of the bullet, expect the parabola of the contour to come to a vertical, flat surface at the nose of the bullet-slope there is undefined)

Alpha is the angle of deflection of the particles off of the bullet. R is the radius, as it varies given the "x" position.

$$KE_{\text{air molecules}} = \frac{1}{2} \rho \int d(\text{volume}) (\text{velocity})^2$$

We'll substitute in our results for Velocity of air molecules and volume

$$\text{Velocity} = \text{velocity}(\text{initial}) \{ -\cos 2\alpha \text{ (in the "x" direction)} + \sin 2\alpha \text{ (in the "y" direction)} \}$$

Velocity(with respect to the lab) = velocity of air molecules with respect to the bullet + velocity of the bullet

$$\text{Velocity}(\text{with respect to the lab}) = \text{velocity}(\text{initial}) [-\cos 2\alpha \text{ ("x" direction)} + \sin 2\alpha \text{ ("y" direction)}] + \text{velocity initial (in the "x" direction)}$$

$$\text{Velocity of the air molecules} = \text{velocity}(\text{initial}) \{ (1 - \cos 2\alpha) \text{ ("x" direction)} + \sin 2\alpha \text{ ("y" direction)} \}$$

$$\text{Velocity of air molecule} = 2(\text{velocity}) \sin \alpha$$

$$KE_{\text{air molecules}} = (2\pi/2) \rho \text{ velocity}(\text{initial}) \Delta(\text{time}) \int f * f' dx (2\text{velocity}(\text{initial}) \sin 2\alpha)^2$$

Bla, bla, bla.....my partner and our professor got us down to a final differential form of

$$\frac{\partial}{\partial x} [r / (1 + x'^2)] - (d/dr) (\frac{\partial}{\partial x}) [r / (1 + x'^2)]$$

This part-- $\frac{\partial}{\partial x} [r / (1 + x'^2)]$ -- goes to zero.

Since $[r / (1 + x'^2)]$ is ultimately our "F" function for the Euler-Lagrange equation

$$\frac{\partial F}{\partial y} - d/dx (\frac{\partial F}{\partial y'}) = 0$$

We're left with the special case where our Euler-Lagrange equation can be reduced to

$$- d/dx (\frac{\partial F}{\partial y'}) = 0$$

so, $(\frac{\partial F}{\partial y'})$ must equal a constant

In terms of our variables we have:

$$0 - d/dr (\frac{\partial}{\partial x'}) [r / (1 + x'^2)] \rightarrow -d/dx (\frac{\partial}{\partial y'}) [x / (1 + y'^2)] \text{ so, our F is not dependent upon y, only } y'.$$

What next?! !!!