This question is about length contraction.

(a) In reference frame S, the two ends of a stationary rod have space time coordinates at the origin $(0, 0, 0; 0)$ and the point $(L, 0, 0; t)$. In another reference frame S', moving at speed u in the x-direction with respect to frame S, these two points are $(0, 0, 0; 0)$ and $(L', 0, 0; t')$ respectively.

(i) Explain why L is the length of the rod in frame S, no matter what the value of t is; but t' must be zero if we wish to interpret L' as the length of the rod in frame S'.

(ii) Hence from the Lorentz transformation (see appendix), derive the expression for length contraction relating L' to L.

(iii) Which of these two lengths will be larger?

(b) With the same notation as part (a), if $t' = 0$, find t in terms of L and/or L', and hence show that

$$L'^2 = L^2 - c^2t^2.$$