



For $t = 0$ sec $u_0 = 0 \frac{m}{s}$

T : Fluid friction

Buoyancy = 0

(a) $u(t) = ?$

(b) limit speed = ?

$$\left. \begin{array}{l} \Sigma F = ma \\ \Sigma F = W - T \end{array} \right\} ma = mg - kv^2 \Rightarrow a = g - \frac{k}{m} v^2 \Rightarrow$$

$$\Leftrightarrow \frac{\partial u}{\partial t} = g - \frac{k}{m} u^2 \Rightarrow \frac{1}{g - \frac{k}{m} u^2} \partial u = \partial t \Leftrightarrow$$

$$\Leftrightarrow -\frac{u}{k} \frac{1}{u^2 - \frac{gu}{k}} \partial u = \partial t \Leftrightarrow -\frac{u}{k} \int_0^u \frac{1}{u^2 - \frac{gu}{k}} \partial u = \int_0^t \partial t \Leftrightarrow$$

$$\Leftrightarrow -\frac{u}{k} \int_0^u \frac{1}{\left(u - \sqrt{\frac{gu}{k}}\right) \left(u + \sqrt{\frac{gu}{k}}\right)} \partial u = \int_0^t \partial t \quad (*)$$

$$\bullet \frac{1}{\left(u - \sqrt{\frac{gu}{k}}\right) \left(u + \sqrt{\frac{gu}{k}}\right)} = \frac{A}{u - \sqrt{\frac{gu}{k}}} + \frac{B}{u + \sqrt{\frac{gu}{k}}} \Leftrightarrow$$

$$\Leftrightarrow 1 = A \left(u + \sqrt{\frac{gu}{k}}\right) + B \left(u - \sqrt{\frac{gu}{k}}\right)$$

$$\text{For } u = \sqrt{\frac{gu}{k}} \rightsquigarrow A = \frac{1}{2\sqrt{\frac{gu}{k}}}$$

$$\text{For } u = -\sqrt{\frac{gu}{k}} \rightsquigarrow B = -\frac{1}{2\sqrt{\frac{gu}{k}}}$$

$$\Leftrightarrow -\frac{u}{k} \int_0^u \left(\frac{1}{2\sqrt{\frac{gu}{k}} \left(u - \sqrt{\frac{gu}{k}} \right)} - \frac{1}{2\sqrt{\frac{gu}{k}} \left(u + \sqrt{\frac{gu}{k}} \right)} \right) du = \int_0^t dt \quad (\Leftrightarrow)$$

$$\Leftrightarrow -\frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \int_0^u \frac{1}{u - \sqrt{\frac{gu}{k}}} du + \frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \int_0^u \frac{1}{u + \sqrt{\frac{gu}{k}}} du = \int_0^t dt \quad (\Leftrightarrow)$$

$$\Leftrightarrow -\frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \left(\ln \left| u - \sqrt{\frac{gu}{k}} \right| \right)_0^u + \frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \left(\ln \left| u + \sqrt{\frac{gu}{k}} \right| \right)_0^u = t \quad (\Leftrightarrow)$$

$$\Leftrightarrow -\frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \left(\ln \left| u - \sqrt{\frac{gu}{k}} \right| - \ln \left| -\sqrt{\frac{gu}{k}} \right| \right) + \frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \left(\ln \left| u + \sqrt{\frac{gu}{k}} \right| - \ln \left| \sqrt{\frac{gu}{k}} \right| \right) = t \Leftrightarrow$$

$$\Leftrightarrow -\frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \ln \left| \frac{u - \sqrt{\frac{gu}{k}}}{-\sqrt{\frac{gu}{k}}} \right| + \frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \ln \left| \frac{u + \sqrt{\frac{gu}{k}}}{\sqrt{\frac{gu}{k}}} \right| = t \Leftrightarrow$$

$$\Leftrightarrow \frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}} \left(\ln \left| \frac{u + \sqrt{\frac{gu}{k}}}{\sqrt{\frac{gu}{k}}} \right| - \ln \left| \frac{u - \sqrt{\frac{gu}{k}}}{-\sqrt{\frac{gu}{k}}} \right| \right) = t \Leftrightarrow$$

$$\Leftrightarrow \ln \left| \frac{u + \sqrt{\frac{gu}{k}}}{\sqrt{\frac{gu}{k}}} \right| - \ln \left| \frac{u - \sqrt{\frac{gu}{k}}}{-\sqrt{\frac{gu}{k}}} \right| = \frac{t}{\frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}}} \quad \Leftrightarrow$$

$$\Leftrightarrow \ln \left| \frac{\frac{u + \sqrt{\frac{gu}{k}}}{\sqrt{\frac{gu}{k}}}}{\frac{u - \sqrt{\frac{gu}{k}}}{-\sqrt{\frac{gu}{k}}}} \right| = \frac{t}{\frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}}} \quad \Leftrightarrow$$

$$\Leftrightarrow \frac{\frac{u + \sqrt{\frac{gu}{k}}}{\sqrt{\frac{gu}{k}}}}{\frac{u - \sqrt{\frac{gu}{k}}}{-\sqrt{\frac{gu}{k}}}} = e^{\left(\frac{t}{\frac{u}{k} \frac{1}{2\sqrt{\frac{gu}{k}}}} \right)} \quad \Leftrightarrow$$

$$\frac{\left(u + \sqrt{\frac{g_{44}}{k}}\right) \left(-\sqrt{\frac{g_{44}}{k}}\right)}{\left(u - \sqrt{\frac{g_{44}}{k}}\right) \left(\sqrt{\frac{g_{44}}{k}}\right)} = e^{\left(\frac{t}{\frac{u}{k} \frac{1}{2\sqrt{\frac{g_{44}}{k}}}}\right)} \quad (\Rightarrow)$$

$$(\Rightarrow) - \frac{u + \sqrt{\frac{g_{44}}{k}}}{u - \sqrt{\frac{g_{44}}{k}}} = e^{\left(\frac{t}{\frac{u}{k} \frac{1}{2\sqrt{\frac{g_{44}}{k}}}}\right)} \quad (\Rightarrow)$$

$$(\Rightarrow) -u + \sqrt{\frac{g_{44}}{k}} = e^{(\dots)} \left(u - \sqrt{\frac{g_{44}}{k}}\right) \quad (\Rightarrow) -u + \sqrt{\frac{g_{44}}{k}} = e^{(\dots)} \cdot u - e^{(\dots)} \cdot \sqrt{\frac{g_{44}}{k}} \quad (\Rightarrow)$$

$$\Rightarrow e^{(\dots)} \cdot u + u = e^{(\dots)} \cdot \sqrt{\frac{gW}{k}} + \sqrt{\frac{gW}{k}} \Rightarrow u(e^{(\dots)} + 1) = (e^{(\dots)} + 1) \sqrt{\frac{gW}{k}} \Rightarrow$$

$$\boxed{u = \sqrt{\frac{gW}{k}}} \quad \leadsto \quad u = \sqrt{\frac{W}{k}}$$