

**16. Let  $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$ . Let**

$$L^2(D) = \{f: D \rightarrow \mathbb{C} \mid \iint |f(x, y)|^2 dx dy < \infty\}$$

**Define an inner product on  $L^2(D)$  by**

$$\langle f, g \rangle = \iint f(x, y) \overline{g(x, y)} dx dy$$

**Let  $\phi_n(x, y) = (x + iy)^n$ ,  $n = 0, 1, 2, \dots$ . Show that this collection of functions is orthogonal in  $L^2(D)$  and compute  $\|\phi_n\|$ .**

Let  $\phi_j, \phi_k \in L^2(D)$ , then

$$\langle \phi_j, \phi_k \rangle = \iint \phi_j(x, y) \overline{\phi_k(x, y)} dx dy = \iint (x + iy)^j (x - iy)^k dx dy = \iint (x^{j+k} + y^{j+k}) dx dy$$