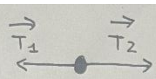

$\sum \vec{F}_{ext} = m\vec{a}$ (Newton's 2nd law)

x is the position of the point mass
$\dot{x}$ is the velocity of the point mass
$\ddot{x}$ is the acceleration of the point mass

Using the above notation, Newton's 2^{nd} law can be re-written as:

$$m\ddot{x} = U_1 T_1 + U_2 T_2$$

defining $\underline{U} \equiv \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$, $m\ddot{x} = \begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$

Since a state space representation involves first derivatives only, we pose:

$x_1 = x$
$x_2 = \dot{x}$

This allows to rewrite Newton's 2^{nd} law as:

$$\begin{cases} \dot{x}_2 = \begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \\ \dot{x}_1 = x_2 \end{cases}$$

, or, in matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \emptyset & 1 \\ \emptyset & \emptyset \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{T_1}{m} & \frac{T_2}{m} \\ \emptyset & \emptyset \end{bmatrix}}_{B} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

The problem with the B matrix just defined is that A is a $n \times n$ with $n = 2$ matrix, but B is a $n \times m$ with $n = 2$, $m = 2$

So when testing for controllability, we are going to have to compute $\mathrm{Rank}\left(\underbrace{\left[B \,|\, AB \,|\, A^2B \,|\, \cdots A^{n-1}B \right]}_{C}\right)$

and check if $\mathrm{Rank}(C) = n$

given the fact that $n = 2$, $C = [B \,|\, AB]$

$$A \cdot B = \begin{bmatrix} \emptyset & 1 \\ \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} \frac{T_1}{m} & \frac{T_2}{m} \\ \emptyset & \emptyset \end{bmatrix} = \begin{bmatrix} \emptyset & \emptyset \\ \emptyset & \emptyset \end{bmatrix}$$ and $C = \begin{bmatrix} \frac{T_1}{m} & \frac{T_2}{m} & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset \end{bmatrix}$

C is not square, so there is no determinant for C which prevents answering the question "is the system controllable?"

So the state space formulation previously presented needs to be re-worked.

Clearly, the reason for B being n by m, where $n = m = 2$ is because of the input vector $\underline{U} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$, so we <u>cheat</u>, and assume that the two separate control inputs U_1 and U_2 are lumped into "control mixer" $\underline{U}$ where $\underline{U}$ is a scalar.

This assumption allows re-writing the B matrix of the dynamic system as $B = \begin{bmatrix} \frac{T_1 - T_2}{m} \\ \emptyset \end{bmatrix}$, and the newly formulated state space representation becomes:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \emptyset & 1 \\ \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{T_1 - T_2}{m} \\ \emptyset \end{bmatrix} u$$

In turn, $AB = \begin{bmatrix} \emptyset & 1 \\ \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} \frac{T_1 - T_2}{m} \\ \emptyset \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$, and $C = [B \,|\, AB] = \begin{bmatrix} \frac{T_1 - T_2}{m} & \emptyset \\ \emptyset & \emptyset \end{bmatrix}$

If we pose $\underline{V_1} = \begin{bmatrix} \frac{T_1 - T_2}{m} \\ \emptyset \end{bmatrix}$, $\underline{V_2} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$, $\underline{V_2} = \emptyset\, \underline{V_1}$, so the two vectors $\underline{V_1}$ and $\underline{V_2}$ are not linearly independant, so $\mathrm{Rank}(C) = 1$ so the dynamic system is not fully controllable, so there is no point trying to control it with an LQR approach.