Question # 2

The number of customers at Winkies Donuts between 8:00a.m. and 9:00a.m. is believed to follow a Poisson distribution with a mean of 2 customers per minute. During a randomly selected one minute interval during this time period, what is the probability of 6 customers arriving to Winkies? What is the probability that at least 2 minutes elapse between customer arrivals?

Answers

1. *Here we apply Poisson distribution because the distribution is over as given period of time. The stated mean of the problem is 2.*

   The formula is

   $$P(X) = \frac{\mu^X e^{-\mu}}{X!}$$

   Where the random variable X as the number of customers arriving in the interval and μ is the mean.

   Substituting values:

   $$P(6) = \frac{(2^6 e^{-2})}{(6!)}$$

   $$\frac{64 \times .135}{720} = \underline{.012}$$

   So, there is 1.2% probability of 6 customers arriving to Winkies in a randomly selected one minute interval.

2. *Next we have to find the probability of 2 minute elapse between customer arrivals. Currently 2 customers per minute so a 2 minutes gap can be represented as 2/3 customers per minute.*

   Using exponential probability formula

   $$P(x < x_0) = 1 - e^{-(x_0/\mu)}$$

   $$P(x \geq 2) = 1 - e^{-(2\times1)}$$

   $$1 - 0.71828 = 0.28172$$

There fore, 28.17 % is the probability of 2 minutes gap between customer arrivals.

Question # 3

A large consignment of tennis balls is assumed to have 20% substandard balls with standard deviation of 5. A sample of 400 balls is selected. Find the probability that percentage of substandard balls in the sample is

i) at most 16% ii) at least 22% iii) between 18% and 22%

Answers

1. *Let* $\sigma = 5$, μ = 20% of 400 = 80 and x = 15.9% of 400 = 63.6.

   $P(x<=16)$

   $= P(x<15.9)$

   $z = (x - \mu)/\sigma$

   $z = (63.6-80)/5 = -3.28$

   $P(z<-3.28)$

   $=\underline{.0005}$

2. *Let* $\sigma = 5$, μ = 20% of 400 = 80 and x = 22.1% of 400 = 88.4

   $P(x>=22)$

   $= P(x>22.1)$

   $z = (x - \mu)/\sigma$

   $z = (88.4-80)/5 = 1.68$

   $P(z>1.68)$

   $=P(z<-1.68)$

   $=\underline{.0465}$

3. *Let* $\sigma = 5$, μ = 20% of 400 = 80 and x between 18% of 400 = 72 and 22% of 400 = 88

P(74< x <88)

$z = (x - \mu)/\sigma$

z1 = (74-80)/5 = -1.2

z2 = (88-80)/5 = 1.6

P(-1.2 < z < 1.6)

= P(z<1.6) – P(z<-1.2)

= .9452 - .1151

= <u>0.8301</u>

Question # 4

The following probability model describes the number of snow storms for Washington, D. C. for a given year:

# of Snowstorms	0	1	2	3	4	5	6
Probability	.25	.33	.24	.11	.04	.02	.01

The probability of 7 or more snowstorms in a year is 0.

a. What is the probability of more than 2 but less than 5 snowstorms?

b. Given this a particularly cold year (in which 2 snowstorms have already been observed), what is the conditional probability that 4 or more snowstorms will be observed?

c. If at the beginning of winter there is a snowfall, what is the probability of at least one more snowstorm before winter is over?

Answers

a. *We can phrase the question as*

   $P\ (2 < X <= 4)$

   $= 0.11 + 0.04$

   $= \underline{0.15}$

b. *Here we have to find the conditional probability of 4 or more given 2 has occurred.*

   $P\ (X>=4|X=2)$

   $P\ (X>=4) = .04 + .02 + .01$
   $= 0.07$
   $P(X=2) = 0.24$

   $P\ (X>=4|X=2) = .07/.24$
   $= \underline{0.29}$

c. *The question asks the conditional probability of 1 more given 1 has occurred.*

   $P\ (X=2 \mid X=1)$

   $P\ (X=2) = .24$

   $P(X=1) = 0.33$

   $P\ (X=2|X=1)$

   $= .24/.33$

   $= 0.727$

   $\underline{\sim}\ \underline{0.73}$