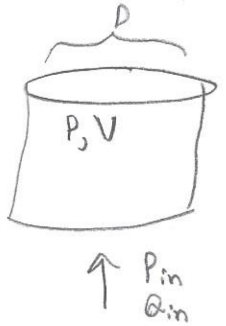
8.



$V_{tank}(0) = V_0$

$P(0) = P_0$

$Q_{in} = \alpha(P_{in} - P)$

isothermal $\Rightarrow$ T constant

a) The inert gas will push against the fluid and will prevent it from completely filling up the tank.

b) The final pressure must be $\boxed{P = P_{in}}$ so that no more fluid comes into the tank

c)

Need an expression for $P(V_\ell)$, where V is the volume occupied by the liquid

$$\frac{dm}{dt} = \rho Q_{in} = \rho\,\alpha(P_{in} - P)$$

$$V_{gas}(t) = V_0 - Q_{in} t$$

$$PV_{gas} = nRT$$

$$V_{gas}(t) = V_0 - \alpha(P_{in} - P)t$$

$$V_\ell(t) = Q_{in} t = \alpha(P_{in} - P)t$$

I need an expression where $P(0) = P_0$ & $P(t \to \infty) = P_{in}$

Not sure how to "perform a mass balance" on the gas, it's mass in the tank never changes, or how doing a mass balance on the gas will aid in solving the problem.

$$\frac{dm_{gas}}{dt} = \frac{d(\rho_{gas} V_{gas})}{dt} = \rho_{gas}\frac{dV_{gas}}{dt} + V_{gas}\frac{d\rho_{gas}}{dt}$$

$$V_{gas} = V_0 - \alpha(P_{in} - P)t \qquad \Downarrow$$

$$-\rho_{gas}\alpha(P_{in} - P) + \left[V_0 - \alpha(P_{in} - P)t\right]\frac{d\rho_{gas}}{dt}$$

$$\frac{dV_{gas}}{dt} = -\alpha(P_{in} - P) \qquad \Rightarrow -\rho_{gas}\alpha(P_{in} - P) + \left[V_0 - \cancel{\alpha(P_{in} - P)}t\right] \cdot \frac{-n}{\left[V_0 - \alpha(P_{in} - P)t\right]^2}$$

How to get an expression for $\rho_{gas}(t)$?

$$\rho_{gas} = \frac{n}{V_{gas}} = \frac{n}{V_0 - \alpha(P_{in} - P)t} \qquad \frac{d\rho_{gas}}{dt} = \frac{d}{dt}\left[\frac{n}{V_0 - \alpha(P_{in} - P)t}\right]$$

$$n\left[\ldots\right]^{-1}$$