Problem n. 7 chapter 4 Eisberg Resnick ”Quantum Physics”

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Show that the number of α particles scattered by an angle Θ or greater in Rutherford scattering is

$(\frac{1}{4\pi\epsilon_0})^2\pi I\rho t(\frac{zZe^2}{Mv^2})^2\cot^2(\Theta/2)$ (1)

SOLUTION

starting from Rutherford formula dN=$(\frac{1}{4\pi\epsilon_0})^2\pi I\rho t(\frac{zZe^2}{Mv^2})^2\frac{1}{\sin^4(\Theta/2)}d\Omega$ (1)

$d\Omega = 2\pi\sin(\Theta)d\Theta$

integrate (1) from Θ to π

N=$(\frac{1}{4\pi\epsilon_0})^2\pi I\rho t(\frac{zZe^2}{Mv^2})^2\int_\Theta^\pi\frac{2\pi\sin(\Theta)d\Theta}{\sin^4(\Theta/2)}$

let u=$\Theta/2$; $d\Theta = 2du$; $\sin(\Theta) = 2\sin(\Theta/2)\cos(\Theta/2)$

=$(\frac{1}{4\pi\epsilon_0})^2 8\pi^2 I\rho t(\frac{zZe^2}{Mv^2})^2\int\frac{\sin(u)\cos(u)}{\sin^4(u)}du$

=$(\frac{1}{4\pi\epsilon_0})^2 8\pi^2 I\rho t(\frac{zZe^2}{Mv^2})^2\int\frac{\cos(u)}{\sin^3(u)}du$

=$(\frac{1}{4\pi\epsilon_0})^2 8\pi^2 I\rho t(\frac{zZe^2}{Mv^2})^2\int\frac{d\sin(u)}{\sin^3(u)}$

=$(\frac{1}{4\pi\epsilon_0})^2 8\pi^2 I\rho t(\frac{zZe^2}{Mv^2})^2(-\frac{1}{2})\frac{1}{\sin^2(\Theta/2)}\;|_\Theta^\pi$

=$(\frac{1}{4\pi\epsilon_0})^2 8\pi^2 I\rho t(\frac{zZe^2}{Mv^2})^2(-\frac{1}{2})(1-\frac{1}{\sin^2(\Theta/2)})$

=$(\frac{1}{4\pi\epsilon_0})^2 8\pi^2 I\rho t(\frac{zZe^2}{Mv^2})^2(\frac{1}{2})(\frac{1-sin^2(\Theta/2}{\sin^2(\Theta/2)})$

=$\frac{1}{4\epsilon_0^2}I\rho t(\frac{zZe^2}{Mv^2})^2\cot^2(\Theta/2)$

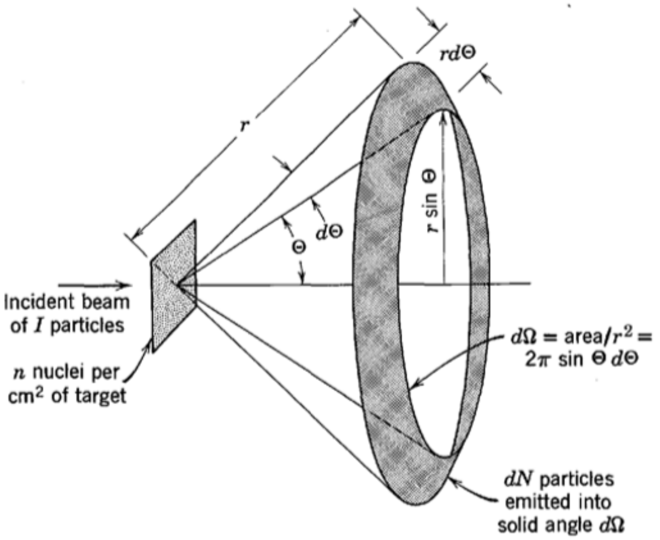

Figure 4-8 Illustrating the definition of the differential cross section $d\sigma/d\Omega$. If the target is thin enough for an incident particle to have negligible chance of interacting with more than one nucleus while passing through the target, then $dN = (d\sigma/d\Omega)In\, d\Omega$