Prove: If $c \mid ab$ and $(c, a) = d$, then $c \mid db$.

Proof: Suppose $c \mid ab$ and $(c, a) = d$.
Since $\gcd(c, a) = d$, it follows that $cu + av = d$ for some integers u and v.
Multiplying this equation by b produces $bcu + abv = db$.
Note that $c \mid ab$.
Thus $ab = ct$ for some $t \in \mathbb{Z}$.
Inserting this expression for ab into $bcu + abv = db$ produces
$bcu + ctv = db$,
or $c(bu + tv) = db$, and so $c \mid db$.
Therefore, if $c \mid ab$ and $(c, a) = d$, then $c \mid db$.