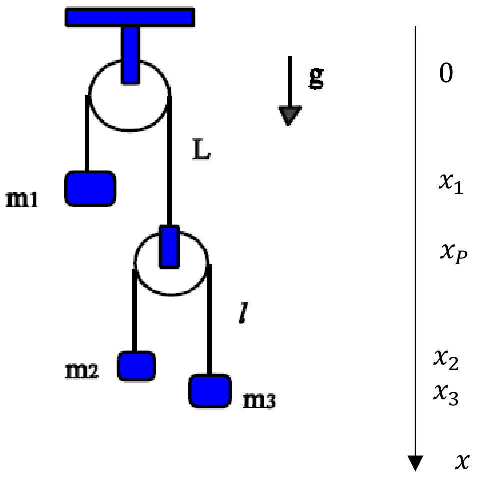

4 particles

Constraints: They only move vertically

2 inextensible threads

Degrees of freedom: 2

$$\begin{cases} L = x_1 + x_P \\ l = x_2 + x_3 \pm 2x_P \end{cases} \Rightarrow \begin{cases} 0 = \ddot{x}_1 + \ddot{x}_P \\ 0 = \ddot{x}_2 + \ddot{x}_3 - 2\ddot{x}_P \end{cases} \Rightarrow -\ddot{x}_P = \ddot{x}_1 = -\frac{\ddot{x}_2 + \ddot{x}_3}{2} \qquad (0)$$

a) Using Newton's equations:

$$m_1\, g - T_L = m_1\, \ddot{x}_1 \qquad \Rightarrow 2T_l - m_1\, g = \frac{m_1}{2}(\ddot{x}_2 + \ddot{x}_3) \qquad (1)$$

$$m_2\, g - T_l = m_2\, \ddot{x}_2 \qquad (2)$$

$$m_3\, g - T_l = m_3\, \ddot{x}_3 \qquad (3)$$

$$2T_l - T_L = 0 \qquad \Rightarrow \quad 2T_l = T_L$$

From (2):

$$T_l = m_2(g - \ddot{x}_2)$$

In (3):

$$m_3\, g - m_2(g - \ddot{x}_2) = m_3\, \ddot{x}_3$$

$$\ddot{x}_3 = g - \frac{m_2}{m_3}(g - \ddot{x}_2)$$

$$\ddot{x}_3 = \left(1 - \frac{m_2}{m_3}\right) g + \frac{m_2}{m_3}\ddot{x}_2 \qquad (4)$$

In (1):

$$2m_2(g - \ddot{x}_2) - m_1\, g = \frac{m_1}{2}\left[\left(1 - \frac{m_2}{m_3}\right) g + \left(1 + \frac{m_2}{m_3}\right)\ddot{x}_2\right]$$

$$4m_2 g - 4m_2\ddot{x}_2 - 2m_1\, g = m_1\left(1 - \frac{m_2}{m_3}\right) g + m_1\left(1 + \frac{m_2}{m_3}\right)\ddot{x}_2$$

$$\left[m_1\left(1 + \frac{m_2}{m_3}\right) + 4m_2\right]\ddot{x}_2 = \left[4m_2 - 2m_1 - m_1\left(1 - \frac{m_2}{m_3}\right)\right] g$$

$$\ddot{x}_2 = \frac{4m_2 - m_1\left(3 - \frac{m_2}{m_3}\right)}{4m_2 + m_1\left(1 + \frac{m_2}{m_3}\right)}$$

$$\ddot{x}_2 = \frac{4m_2 m_3 - m_1(3m_3 - m_2)}{4m_2 m_3 + m_1(m_3 + m_2)}$$

In (4):

$$\ddot{x}_3 = \left[1 - \frac{m_2}{m_3} + \frac{m_2}{m_3}\,\frac{4m_2 m_3 - m_1(3m_3 - m_2)}{4m_2 m_3 + m_1(m_3 + m_2)}\right] g$$

$$\ddot{x_3} = \frac{m_3[4m_2m_3 + m_1(m_3 + m_2)] - m_2[4m_2m_3 + m_1(m_3 + m_2)] + m_2[4m_2m_3 - m_1(3m_3 - m_2)]}{m_3[4m_2m_3 + m_1(m_3 + m_2)]} g$$

$$\ddot{x_3} = \frac{4m_2m_3^2 + m_1m_3(m_3 + m_2) - m_1m_2(m_3 + m_2) - m_1m_2(3m_3 - m_2)}{m_3[4m_2m_3 + m_1(m_3 + m_2)]} g$$

$$\ddot{x_3} = \frac{4m_2m_3^2 + m_1m_3(m_3 + m_2) - m_1m_2(m_3 + m_2) - m_1m_2(3m_3 - m_2)}{4m_2m_3^2 + m_1m_3^2 + m_1m_2m_3} g$$

$$\ddot{x_3} = \frac{4m_2m_3^2 + m_1(m_3^2 + m_2m_3 - m_2m_3 - m_2^2 - 3m_2m_3 + m_2^2)}{4m_2m_3^2 + m_1m_3^2 + m_1m_2m_3} g$$

$$\ddot{x_3} = \frac{4m_2m_3^2 + m_1(m_3^2 - 3m_2m_3)}{4m_2m_3^2 + m_1m_3^2 + m_1m_2m_3} g$$

$$\ddot{x_3} = \frac{(4m_2 + m_1)m_3 - 3m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2} g$$

In (0):

$$\ddot{x_1} = -\frac{1}{2}\left[\frac{4m_2m_3 - m_1(3m_3 - m_2)}{4m_2m_3 + m_1(m_3 + m_2)} + \frac{(4m_2 + m_1)m_3 - 3m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2}\right] g$$

(simplified on part b)

b) Using D'Alembert's principle:

$$\begin{cases} r_1 = L - \dfrac{x_2 + x_3 - l}{2} \\ r_2 = x_2 \\ r_3 = x_3 \end{cases} \Rightarrow \begin{cases} \delta r_1 = -\dfrac{\delta x_2 + \delta x_3}{2} \\ \delta r_2 = \delta x_2 \\ \delta r_3 = \delta x_3 \end{cases}$$

$$\sum_{i=1}^{N} \left(m_i \ddot{\boldsymbol{r}}_i - \boldsymbol{F}_i^{(a)}\right) \cdot \delta \boldsymbol{r}_i = 0$$

$$-m_1(\ddot{x_1} - g)\left(\frac{\delta x_2 + \delta x_3}{2}\right) + m_2(\ddot{x_2} - g)\delta x_2 + m_3(\ddot{x_3} - g)\delta x_3 = 0$$

$$\frac{m_1}{2}(g - \ddot{x_1}) + m_2(\ddot{x_2} - g) = 0 \qquad (5)$$

$$\frac{m_1}{2}(g - \ddot{x_1}) + m_3(\ddot{x_3} - g) = 0 \qquad (6)$$

From (0) in (5):

$$\frac{m_1}{2}\left(g + \frac{\ddot{x_2} + \ddot{x_3}}{2}\right) + m_2(\ddot{x_2} - g) = 0$$

$$\left(\frac{m_1}{2} - m_2\right) g + \left(\frac{m_1}{4} + m_2\right)\ddot{x_2} + \frac{m_1}{4}\ddot{x_3} = 0$$

$$(2m_1 - 4m_2)g + (m_1 + 4m_2)\ddot{x_2} + m_1\ddot{x_3} = 0$$

$$\ddot{x_2} = \frac{(4m_2 - 2m_1)g - m_1\ddot{x_3}}{m_1 + 4m_2} \qquad (7)$$

In (6):

$$\frac{m_1}{2}\left(g + \frac{\ddot{x_2} + \ddot{x_3}}{2}\right) + m_3(\ddot{x_3} - g) = 0$$

$$\frac{m_1}{2}\left(g + \frac{(4m_2 - 2m_1)g - m_1\ddot{x_3}}{2m_1 + 8m_2} + \frac{\ddot{x_3}}{2}\right) + m_3(\ddot{x_3} - g) = 0$$

$$\frac{m_1}{2}g + \frac{m_1(2m_2 - m_1)}{2m_1 + 8m_2}g - \frac{m_1^2}{4m_1 + 16m_2}\ddot{x_3} + \frac{m_1}{4}\ddot{x_3} + m_3\ddot{x_3} - m_3 g = 0$$

$$\left[\frac{m_1}{4} + m_3 - \frac{m_1^2}{4m_1 + 16m_2}\right]\ddot{x_3} + \left[\frac{m_1}{2} - m_3 + \frac{m_1(2m_2 - m_1)}{2m_1 + 8m_2}\right]g = 0$$

$$\frac{m_1(16m_2 + 4m_1) + 4m_3(16m_2 + 4m_1) - 4m_1^2}{16(4m_2 + m_1)}\ddot{x_3} + \frac{m_1(2m_1 + 8m_2) - 2m_3(2m_1 + 8m_2) + 2m_1(2m_2 - m_1)}{4(4m_2 + m_1)}g = 0$$

$$\ddot{x_3} = -\frac{m_1(2m_1 + 8m_2) - 2m_3(2m_1 + 8m_2) + 2m_1(2m_2 - m_1)}{4(4m_2 + m_1)} \frac{16(4m_2 + m_1)}{m_1(16m_2 + 4m_1) + 4m_3(16m_2 + 4m_1) - 4m_1^2} g$$

$$\ddot{x_3} = -\frac{m_1(m_1 + 4m_2) - m_3(2m_1 + 8m_2) + m_1(2m_2 - m_1)}{2} \frac{4}{m_1(4m_2 + m_1) + m_3(16m_2 + 4m_1) - m_1^2} g$$

$$\ddot{x_3} = -\frac{m_1^2 + 4m_1m_2 - 2m_1m_3 - 8m_2m_3 + 2m_1m_2 - m_1^2}{2} \frac{4}{4m_1m_2 + m_1^2 + 16m_2m_3 + 4m_1m_3 - m_1^2} g$$

$$\ddot{x_3} = \frac{2m_1m_3 + 8m_2m_3 - 6m_1m_2}{2} \frac{4}{4m_1m_2 + 16m_2m_3 + 4m_1m_3} g$$

$$\ddot{x_3} = \frac{m_1m_3 + 4m_2m_3 - 3m_1m_2}{m_1m_2 + 4m_2m_3 + m_1m_3} g$$

$$\ddot{x_3} = \frac{(4m_2 + m_1)m_3 - 3m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2} g$$

In (7):

$$\ddot{x_2} = \frac{4m_2 - 2m_1}{m_1 + 4m_2}g - \frac{m_1}{m_1 + 4m_2} \frac{(4m_2 + m_1)m_3 - 3m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2} g$$

$$\ddot{x_2} = \frac{(4m_2 - 2m_1)[(4m_2 + m_1)m_3 + m_1m_2] - m_1[(4m_2 + m_1)m_3 - 3m_1m_2]}{(m_1 + 4m_2)[(4m_2 + m_1)m_3 + m_1m_2]}g$$

$$\ddot{x_2} = \frac{(4m_2 - 3m_1)(4m_2 + m_1)m_3 + (4m_2 + m_1)m_1m_2}{(m_1 + 4m_2)[(4m_2 + m_1)m_3 + m_1m_2]}g$$

$$\ddot{x_2} = \frac{(4m_2 - 3m_1)m_3 + m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2}g$$

In (0):

$$\ddot{x_1} = -\frac{1}{2}\left[\frac{(4m_2 - 3m_1)m_3 + m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2} + \frac{(4m_2 + m_1)m_3 - 3m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2}\right]g$$

$$\ddot{x_1} = -\frac{1}{2}\frac{(8m_2 - 2m_1)m_3 - 2m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2}g$$

$$\ddot{x_1} = -\frac{(4m_2 - m_1)m_3 - m_1m_2}{(4m_2 + m_1)m_3 + m_1m_2}g$$