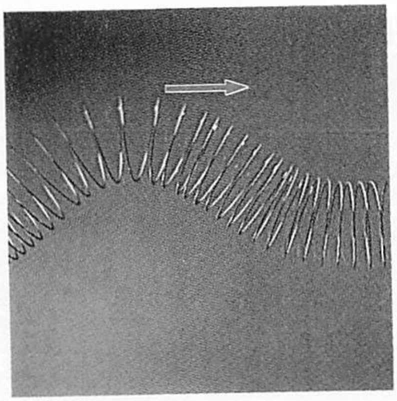

FIGURE 16-12 A pulse on a Slinky is an example of a transverse wave on an elongated structure. The pulse was produced by pulling the Slinky vertically at one point, then releasing.

such case is that of transverse waves on a stretched string, which we're now going to look at in detail. Our results are directly applicable to musical instrument strings, suspension bridge cables, and many other elongated structures (Fig. 16-12).

Our string has mass per unit length μ in kilograms per meter, and is stretched to a tension force F in newtons. In equilibrium, the string lies along the x axis. Suppose we distort a section of it slightly by displacing it in the y direction. We want to show that wave motion results when the string is released and to determine the wave speed.

To find the speed, we need to apply Newton's law to the motion of the string. Figure 16-13*a* shows the wave pulse moving to the right with some speed v. It's easiest to apply Newton's law in a frame of reference moving with the uniform velocity of the pulse; in that frame the entire string is moving to the *left* at speed v. As a section of the string encounters the pulse, which is stationary in this frame of reference, its motion deviates from a straight line as it rides up over the pulse (Fig. 16-13*b*).

Whatever the pulse shape, we can describe a small enough section at the top as a circular arc of some radius R, as shown in Fig. 16-13*c*. Then a small section of string right at the top of the pulse undergoes circular motion with speed v and radius R; if its mass is m, Newton's law requires that a force of magnitude mv^2/R act toward the center of curvature in order to keep the string section on its circular path. This force is provided by the difference in string tension between the two ends of the section; as Fig. 16-13*c* shows, the section's curvature means the tension vectors at the two ends point in different directions. The tension at each end contributes a component $F \sin\theta$ toward the center of curvature, where θ is shown in Fig. 16-13*c*. Then the net force on the segment has magnitude $2F \sin\theta$ and points toward the center of curvature.

Now we make an additional assumption: that the disturbance of the string is small, in the sense that the string remains almost horizontal even at the pulse. Then the angle θ is small, and we can apply the approximation $\sin\theta \simeq \theta$. Therefore the net force on the string section becomes approximately $2F\theta$. Furthermore, the small-disturbance approximation means that the tension doesn't vary significantly from its undisturbed value, so the F in this expression is essentially the same F we're using to characterize the tension throughout the string. Finally, our curved string section forms a circular arc whose length, from Fig. 16-13*c*, is $2\theta R$. Multiplying by the mass per unit length μ gives its mass:

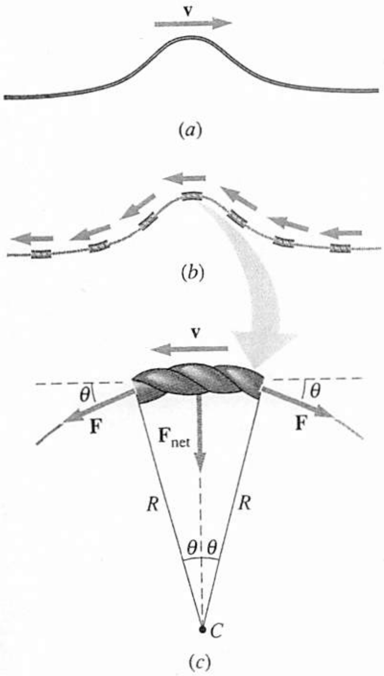

FIGURE 16-13 (*a*) A pulse moving to the right with speed v on a stretched string. (*b*) In the reference frame of the moving pulse, the string moves to the left. Individual segments of the string follow curved paths as they move through the pulse. (*c*) A blow-up of a small string segment at the top of the pulse. The net force on the segment is the vector sum of the tension forces **F** at the two ends; this force keeps the string segment in its path through the curved pulse. The angle θ is exaggerated; it is actually so small that $\sin\theta \simeq \theta$, making $F_{net} \simeq 2F\theta$. The quantity R is the curvature radius at the top of the pulse.

Remember that the analysis of (*c*) is done in a reference frame with the pulse at rest; in that frame, the string really does undergo circular motion at the top of the pulse. But in a reference frame in which the undisturbed string is at rest, the only motion is the back-and-forth motion of the string in the y direction (perpendicular to the string) as the pulse goes by.

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$m = 2\theta R\mu$. Now we can apply Newton's law, equating the net force $2F\theta$ to the mass times acceleration:

$$2F\theta = \frac{mv^2}{R} = \frac{2\theta R\mu v^2}{R} = 2\theta\mu v^2.$$

Solving for the wave speed v then gives

$$v = \sqrt{\frac{F}{\mu}}. \qquad (16\text{-}7)$$

ActivPhysics **10.2** Speed of Waves on a String

Does this result make sense? The string tension provides the restoring force that drives the disturbed part of the string toward equilibrium. The greater the tension F, the greater the acceleration of the disturbed string, and thus the more rapidly the wave should propagate. The string's inertia, on the other hand, limits the acceleration with which the string responds to the force, and therefore a greater mass per unit length should slow the wave propagation. Equation 16-7, with F in the numerator and μ in the denominator, reflects both these trends.

We have made no assumption about wave shape other than to assume that the disturbance is small; therefore Equation 16-7 applies to small-amplitude pulses, continuous waves, and wave trains of any shape. In Section 16-6 we will derive Equation 16-7 using a more advanced analysis that gives a general equation shared by many systems that support propagating waves and will show explicitly that simple harmonic waves are among its solutions.

EXAMPLE 16-2 *Rock Climbing*

Two climbers are joined by a 43-m-long rope of total mass 5.0 kg. One climber strikes the rope with a fist, and 1.4 s later the second climber feels the effect. What is the rope tension?

Solution

Solving Equation 16-7 for the tension gives $F = \mu v^2$. Here $\mu = m/L$ and $v = L/t$, with L and m the rope length and mass and t the time interval. So we have

$$F = \mu v^2 = \left(\frac{m}{L}\right)\left(\frac{L}{t}\right)^2 = \frac{mL}{t^2} = \frac{(5.0\ \text{kg})(43\ \text{m})}{(1.4\ \text{s})^2} = 110\ \text{N}.$$

Is this number reasonable? A typical adult weighs between 500 and 1000 N, so the rope is supporting only a small fraction of the lower climber's weight—a reasonable situation.

EXERCISE A 4.5-g piano wire is under 680 N tension. If waves propagate along it at 320 m/s, what is its length?

Answer: 68 cm

Some problems similar to Example 16-2: 21, 24, 27, 29

16-5 WAVE POWER AND INTENSITY

A wave propagates because part of a medium communicates its motion to adjacent parts. In the process, energy passes through the medium. As always in mechanics, that energy is transferred by forces that do work.

In the case of a stretched string, it is the tension force that does work on the string, and this work results in energy transfer along the string in the direction of the wave propagation. As we showed in Section 7-7, power—the rate of