$$y'(x)+y(x)=\sum_{n=0}^{\infty} a_n n(x-x_0)^{n-1}+a_n(x-x_0)^n$$

$$=\sum_{n=0}^{\infty} a_n(x-x_0)^{n-1}\left(n+(x-x_0)\right)$$

$x_0=1$

$nx_0=n.1=n$ $\quad n(x_0-\frac{1}{n})$

$$=\sum_{n=0}^{\infty} a_n(n-1+x)(x-1)^{n-1}=x^2$$

divide then by x^2 since $x_0=1$ makes the function analytic about x_0

$$\frac{1}{x^2}\left(y'(x)+y(x)\right)=\frac{1}{x^2}\sum_{n=0}^{\infty} a_n(n-1+x)(x-1)^{n-1}=1$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$x-x_0+n=x_0$$

$$(x-x_0+n)(x-x_0)^{n-1}$$

$$(x-x_0-nx_0)(x-x_0)^{n-1}$$

$$\cancel{(x-x_0)}=(x-x_0(1-n)$$

$$=\frac{1}{x^2}\sum_{n=0}^{\infty}\left[a_n n-a_n+a_n x\right](x-1)^{n-1}=1$$

$$a_n\left[(x-1)+n\right]$$