1. $\langle j_1 m_1 | e^{-iHt} | j_2 m_2 \rangle$ is the propagator of eigenstates of $\boldsymbol{J}^2$ and J_z. (Here $\hbar = 1$.)

   (a) Show that when $\left[\boldsymbol{J}^2, H\right] = 0$ this propagator vanishes unless $j_1 = j_2$. We denote in this case $\langle j\, m_1 | e^{-iHt} | j\, m_2 \rangle \equiv K^{(j)}_{m_1 m_2}(t)$.

   (b) Prove that $K^{(j)}_{m_1 m_2}(t) = \sum_m K^{(j)}_{m_1 m}(t - t_1) K^{(j)}_{m m_2}(t_1)$ when $0 < t_1 < t$.

   (c) $H = \omega J_Z$ for a particle in a magnetic field along the z-axis. Find the explicit expression of $K^{(j=1)}_{m_1 m_2}(t)$ for all $m_1 \geq m_2$ for this Hamiltonian (you may use the collection of formulas).

   (d) Calculate $\operatorname{Tr} K^{(j=1)}(t) \equiv \sum_m K^{(j=1)}_{m\, m}(t)$, from the expressions you got in (c).

   (e) Now let $it = \beta$ and reinterpret your result from above as the partition function $Z(\beta) = \operatorname{Tr}(e^{-\beta H})$. What is the expectation value of the energy

$$\langle E \rangle = \operatorname{Tr}\left(\frac{e^{-\beta H}}{Z(\beta)} H \right)$$

   in the zero temperature limit $\beta \to \infty$? Is the result expected?