$$\frac{1}{X}\left(\frac{dX}{dx} - 2xX\right) = \frac{1}{Y}\left(yY - y\frac{dY}{dy}\right) = \lambda$$

$$\frac{1}{X}\frac{dX}{dx} - 2x \qquad\qquad y - \frac{y}{Y}\frac{dY}{dy}$$

$$u = X(x)Y(y)$$

$$\frac{du}{dx} = \frac{dX}{dx}Y$$

$$\frac{du}{dy} = X\frac{dY}{dy}$$

$$\frac{du}{dx} + y\frac{du}{dy} - 2xu - yu = 0$$

$$\frac{dX}{dx}Y + yX\frac{dY}{dy} - 2xXY - yXY = 0$$

$$\frac{1}{X}\frac{dX}{dx} - 2x = \lambda$$

$$\int\left(\frac{1}{X}\frac{dX}{dx} - 2x\right)dx = \int \lambda\, dx$$

$$\int \frac{1}{X}\frac{dX}{dx}dx - \int 2x\, dx = \lambda x$$

$$\int \frac{1}{X}dX - x^2 = \lambda x$$

$$\ln(X) - x^2 = \lambda x$$

$$\ln(X) = \lambda x + x^2$$

$$X(x) = e^{\lambda x + x^2}$$

$$u = \left(e^{\lambda x + x^2}\right)\left(\frac{e^y}{y^\lambda}\right)$$

$$u(x,y) = \frac{e^{x^2 + \lambda x + y}}{y^\lambda}$$

$$y - \frac{y}{Y}\frac{dY}{dy} = \lambda$$

$$y\left(1 - \frac{1}{Y}\frac{dY}{dy}\right) = \lambda$$

$$1 - \frac{1}{Y}\frac{dY}{dy} = \lambda/y$$

$$\frac{1}{Y}\frac{dY}{dy} = 1 - \lambda/y \qquad e^{\ln(x)} = x$$

$$\int \frac{1}{Y}\frac{dY}{dy}dy = \int (1 - \lambda/y)\, dy$$

$$\ln(Y) = y - \lambda \ln(y) = y - \ln(y^\lambda)$$

$$Y(y) = e^{y - \ln(y^\lambda)} = e^y \cdot e^{-\ln(y^\lambda)} = \frac{e^y}{e^{\ln(y^\lambda)}} = \frac{e^y}{y^\lambda}$$