Quantum Foundations V: Relativistic QFT from a Bohmian perspective: A proof of concept

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Part 1.

GENERAL IDEAS

Motivation:

Frequent argument by outsiders:
Bohmian mechanics (BM) is nonlocal.
$\Rightarrow$ BM is not consistent with relativity.
$\Rightarrow$ BM is not consistent with relativistic QFT.
$\Rightarrow$ BM is not consistent with the real world.

The goal is to explain why is that argument wrong!

Since the argument by outsiders is simple,
the counterargument should be simple too
(for otherwise it would not convince many outsiders).

But relativistic QFT is a technically complex theory,
and its Bohmian version is not less complex.
$\Rightarrow$ Fully technical analysis would not be efficient.

$\Rightarrow$ I present a simple conceptual non-technical analysis.
(A proof of concept!)

A simple analogy - classical electromagnetism:

Standard interpretation:
- The “real” ontic stuff is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
- $F_{\mu\nu}$ is **measurable**, **gauge invariant** and **Lorentz covariant**.
- Potential A_μ is not ontic.

Conceptual problems:
- The mathematical formulation of the theory requires A_μ.
- If A_μ is not “real”, it’s hard to understand Aharonov-Bohm effect.

$\Rightarrow$ Alternative interpretation:
- A_μ is a fundamental “real” ontic stuff.

Consequences:
- For given $F_{\mu\nu}$, A_μ is not unique
(due to gauge invariance, A_μ and $A'_\mu = A_\mu + \partial_\mu \lambda$ give the same $F_{\mu\nu}$).
$\Rightarrow$ A_μ can only be ontic if the gauge is completely **fixed**!

The simplest possibility of ontic $A_\mu = (\phi, \mathbf{A})$ is Coulomb gauge:

$$\nabla \cdot \mathbf{A} = 0, \quad \nabla^2 \phi = -\rho$$

$$\phi(\mathbf{x}, t) = \frac{1}{4\pi} \int d^3x' \, \frac{\rho(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}$$

$\Rightarrow$ The ontic theory is **not gauge invariant**, **not Lorentz covariant** and **not local** (instantaneous action at a distance).

- But this theory makes the same **measurable** predictions
as the standard theory
(because mathematically it **is** the standard theory in a fixed gauge).

This is a simple example of the proof of concept:
- An ontic theory which **violates** gauge invariance, Lorentz covariance and locality, yet all its measurable predictions **are** gauge invariant, Lorentz covariant and local.

- Bohmian mechanics for relativistic QFT will turn out to be conceptually similar.

Questions that are hard for (most) physicists, yet easy for philosophers:

Q: What's the **difference** between standard theory and this theory?
A: In this theory A_μ is a "real" ontic stuff.

Q: What's the **point** of saying that A_μ is ontic?
A: The point is mainly conceptual. Computations are the same, but helps to understand some aspects (e.g. Aharonov-Bohm) **intuitively**.
- Ontology is mainly a thinking tool, not a computational tool.
- Those who prefer shut-up-and-calculate may not need this tool at all.

Q: But isn't this example mathematically **trivial**?
A: Yes, but that's exactly the point!
- The point of the example is to realize that something which at first may seem **impossible** (local Lorentz covariant phenomena explained by non-local Lorentz non-covariant theory) may in fact be very **easy**.
- It's a proof of concept!

Arbitrary Lorentz (non)covariant theories, including QFT:

- QFT is "local" in the sense that its Lagrangian density is local.
- We want to understand how measurable predictions of local Lorentz covariant QFT can be explained by a nonlocal Lorentz non-covariant theory such as Bohmian mechanics (BM).

- But we want the argument to be simple, so we don't want to discuss technical details of QFT.

- We do this by making the analysis very **abstract** and **general**.

For a physical theory T, let
$P[T]$ = set of all practically measurable predictions implied by T
- "Practically measurable" refers to measurements that can actually be performed in practice (not merely measurable in principle).

$RQFT$ = Relativistic QFT in the standard (non-Bohmian) formulation.
$RQFT(S)$ = $RQFT$ with calculations performed in a Lorentz frame S.

- The predictions of $RQFT$ are Lorentz invariant:

$$P[RQFT(S)] = P[RQFT(S')] \ \ \forall S, \forall S'.$$

- In words, the measurable predictions do not depend on the Lorentz frame in which the calculations are performed.

BM = Bohmian mechanics (a nonlocal ontic theory of a Bohmian type)
- It's not a relativistic Lorentz covariant theory
(space and time not treated on an equal footing).
- In the language of relativistic theories, one can say that it's formulated in some fixed preferred Lorentz frame S_0.

Now suppose that it is possible to construct a BM theory such that

$$\boxed{P[BM] \supseteq P[RQFT(S_0)]}$$

- In words, we suppose that BM contains all the measurable predictions of $RQFT$ in the Lorentz frame S_0, and, in addition, that BM may contain some additional predictions on which $RQFT$ is silent.

$\Rightarrow$ Lorentz invariance $P[RQFT(S)] = P[RQFT(S')]$ implies

$$P[BM] \supseteq P[RQFT(S)] \ \ \forall S$$

- In other words, if BM reproduces the measurable predictions of $RQFT$ in the preferred frame S_0 (assumption $\boxed{\cdots}$), then it reproduces them in **all** frames S.

- That's how Lorentz non-covariant theory can have Lorentz covariant predictions.

- A nontrivial task is to construct a **concrete** theory with property $\boxed{\cdots}$.

- Actually many different constructions are possible,
some of which we shall discuss next.

Part 2.

A MINIMAL MODEL

Introduction:

- In general we need a BM theory that satisfies $P[BM] \supseteq P[RQFT(S_0)]$
- A minimal model is a theory for which

$$P[BM] = P[RQFT(S_0)]$$

- A minimal model has the same predictions as $RQFT$,
without making any additional predictions.

- As a proof of concept, we shall present the conceptually simple model:
W. Struyve, H. Westman, AIP Conf. Proc. **844**, 321 (2006); quant-ph/0602229

Fundamental Bohmian ontology:

- The only fundamental "real" ontic stuff are bosonic fields,
e.g. Higgs $h(\mathbf{x},t)$, EM potentials $\mathbf{A}(\mathbf{x},t)$ in the Coulomb gauge, etc.
- Particle positions are **not** ontic.
- Fermionic fields are **not** ontic.
- It can be done for the whole Standard Model:
W. Struyve, Rept. Prog. Phys. **73**, 106001 (2010); arXiv:0707.3685

- Intuitively, we usually imagine that the material world is made of charged matter.
- Even though it's not ontic (because it's a fermionic field), charged matter viewed as an effective property of EM field **itself**

$$\rho_{\text{effective charge}}(\mathbf{x},t) \equiv \boldsymbol{\nabla} \cdot \mathbf{E}(\mathbf{x},t)$$

Bohmian ontology for non-relativistic QM:
- a lot of particle positions $\vec{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$

Particle ontology - discrete

Bohmian ontology for QFT:
- fields $\phi(\mathbf{x}) = (h(\mathbf{x}), \mathbf{A}(\mathbf{x}), \ldots)$ at all space points $\mathbf{x}$.

Field ontology - continuous

- At the macroscopic level it looks the same, because in both cases the macroscopic ontology is a pattern in space.

The mathematical construction:

- Main idea is to write QFT in a form similar to non-relativistic QM.
(For details see e.g. B. Hatfield, Quantum Field Theory of Point Particles and Strings (1991).)

Field eigenstates: $\widehat{\phi}(\mathbf{x})|\phi\rangle = \phi(\mathbf{x})|\phi\rangle$.
$\Rightarrow$ All QFT states $|\Psi\rangle$ represented as functionals of bosonic fields

$$\Psi[\phi, t] = \langle\phi|\Psi(t)\rangle$$

- Time evolution given by functional Schrödinger equation

$$\widehat{H}\Psi[\phi, t] = i\hbar\partial_t\Psi[\phi, t]$$

- Relativistic QFT written in a fixed Lorentz frame S_0.
- Does not look manifestly Lorentz covariant.

- The QFT Hamiltonian

$$\hat{H} = \int d^3x \, \frac{\hat{\pi}^2(\mathbf{x})}{2} + V[\phi]$$

where $\hat{\pi}(\mathbf{x}) = -i\hbar \frac{\delta}{\delta\phi(\mathbf{x})}$ is the canonical momentum.

- For comparison, Hamiltonian in non-relativistic QM

$$\hat{H}_{\mathrm{NRQM}} = \sum_{a=1}^{N} \frac{\hat{\mathbf{p}}_a^2}{2m_a} + V(\vec{x})$$

where $\hat{\mathbf{p}}_a = -i\hbar \frac{\partial}{\partial \mathbf{x}_a}$

- Hamiltonians have the same mathematical form, especially if continuous QFT is regularized by a discrete lattice: $\int d^3x \to \sum_{\mathbf{x}}$.

$\Rightarrow$ Bohmian mechanics is a straightforward generalization:
Bohmian particle trajectories $\mathbf{X}_a(t) \to$ Bohmian field trajectories $\Phi(\mathbf{x},t)$

$$\frac{d\mathbf{X}_a(t)}{dt} = \frac{-i\hbar}{2m_a} \frac{\psi^\dagger \overleftrightarrow{\boldsymbol{\nabla}}_a \psi}{\psi^\dagger \psi} \quad \longrightarrow \quad \frac{\partial \Phi(\mathbf{x},t)}{\partial t} = \frac{-i\hbar}{2} \frac{\Psi^\dagger \frac{\overleftrightarrow{\delta}}{\delta\phi(\mathbf{x})} \Psi}{\Psi^\dagger \Psi}$$

Part 3.

TOWARDS A MORE FUNDAMENTAL THEORY

Motivation:

The previous model was minimal: $P[BM] = P[RQFT(S_0)]$
- It assumed that Standard Model is fundamental.

A philosophically unappealing feature:
(1) Measurable predictions (and field action) are Lorentz invariant.
(2) Bohmian equations are not Lorentz covariant.

- It's OK for phenomenology, but it doesn't feel right
for a **really fundamental** theory.
- It would be more elegant if both (1) and (2)
had same symmetries.
- But due to non-locality (Bell theorem), it's hard to make
(2) Lorentz covariant.
$\Rightarrow$ Perhaps it means that (1) is also not Lorentz invariant?

- If so, then **Standard Model is not fundamental**.

$\Rightarrow$ BM gives a hint how to search for a more fundamental theory beyond the Standard Model:

(H.N., Int. J. Quantum Inf. (to appear); arXiv:1811.11643)

- Fundamental action should not be Lorentz invariant.
- Measurable predictions (at very small distances) should not be Lorentz invariant.

We want a fundamental quantum theory F such that

$$P[F] \supseteq P[RQFT(S_0)]$$

and a corresponding Bohmian version of F such that

$$P[BM] = P[F]$$

- To find F is hard, to construct BM (once F is known) is easy.

A promising general framework to find F:
the **condensed matter** framework.

- Standard Model of particle physics is just an **effective** theory,
in the same sense in which field theory of phonons is an effective theory.

- Such models extensively discussed in the literature.

For reviews see the books:

- G.E. Volovik, *The Universe in a Helium Droplet* (Oxford, 2009)
- X.-G. Wen, *Quantum Field Theory of Many-body Systems: From the Origin of Sound to an Origin of Light and Electrons* (Oxford, 2004)

Analogy between phonons and photons:

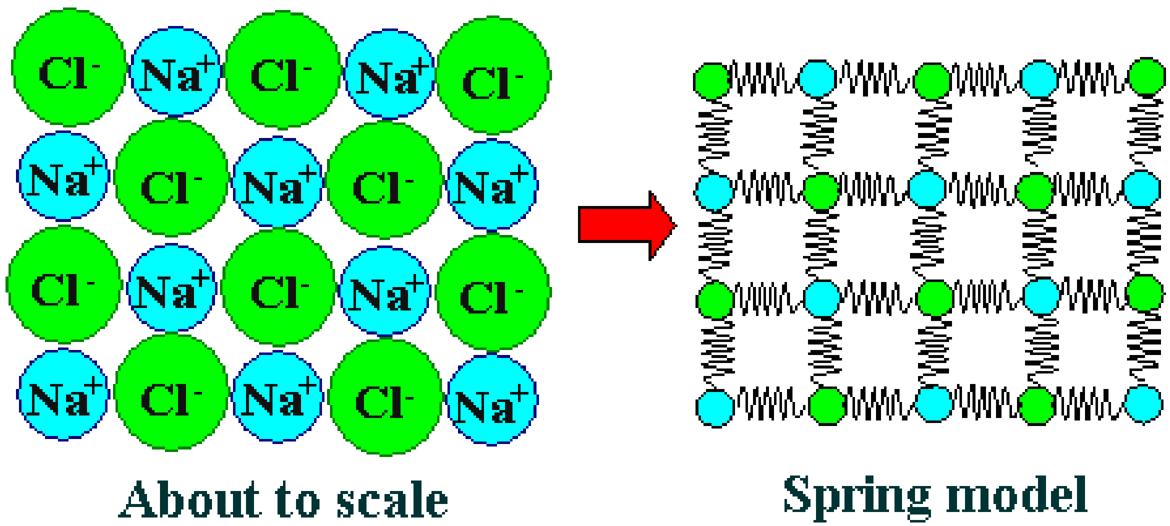
- A phonon is a collective excitation of many particles (atoms).

$\Rightarrow$ From atomic point of view, phonon is a **quasiparticle**, not a "true" particle.

What makes phonons similar to particles?

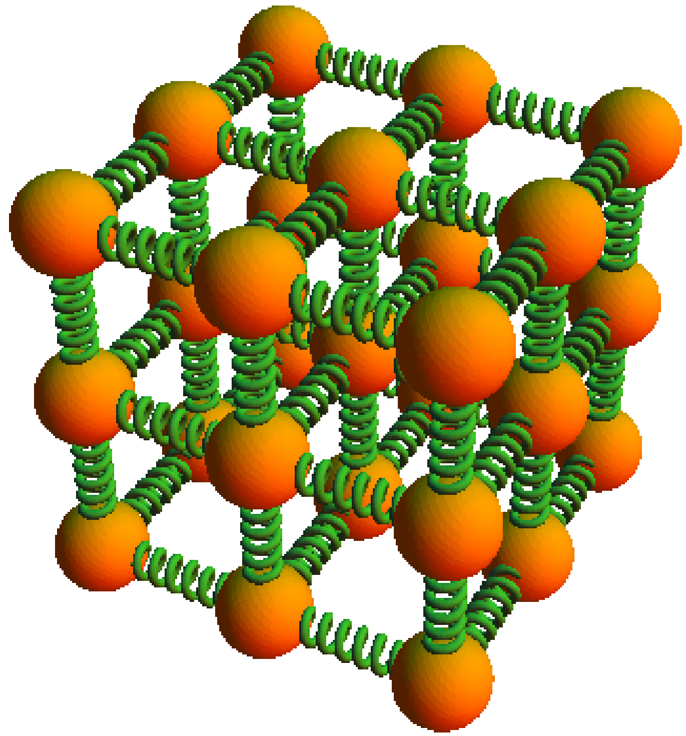
- The key is to approximate the system by a collection of harmonic oscillators.

2-dimensional lattice:



- For each h.o. the potential energy proportional to $(x_i - x_{i+1})^2$.

3-dimensional lattice:



Elementary QM: Each h.o. has energy spectrum of the form

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, 3, \ldots$$

$\Rightarrow$ n can be thought of as a number of “quanta”.

$\Rightarrow$ n behaves like a number of “particles”.

More formally:

- the h.o.'s decouple in new collective coordinates

$k = 1, \ldots, N$ - labels N decoupled harmonic oscillators $\Rightarrow$

$$\hat{H} = \sum_k \hbar\omega_k \left(\hat{n}_k + \frac{1}{2}\right), \quad \hat{n}_k = \hat{a}_k^\dagger \hat{a}_k, \quad [\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'}$$

Complete set of eigenstates:

- groundstate: $|0\rangle$, satisfies $\hat{a}_k|0\rangle = 0$
- 1-"particle" states: $|k\rangle = \hat{a}_k^\dagger|0\rangle$
- 2-"particle" states: $|k_1, k_2\rangle = \hat{a}_{k_1}^\dagger \hat{a}_{k_2}^\dagger|0\rangle$
- 3-"particle" states: ...

- The formalism looks identical to QFT (quantum field theory) of elementary particles (e.g. **photons**).

- Due to this analogy, the above quanta of lattice vibrations are called **phonons**.

$\Rightarrow$ Formally, a phonon is not less a particle than a photon.

- Indeed, photon is also a collective excitation.
- It is a collective excitation of electromagnetic field.
- Electromagnetic field $\mathbf{E}(\mathbf{x})$, $\mathbf{B}(\mathbf{x})$ lives on a continuum 3d space, which can be thought of as a 3d lattice with spacing $l \to 0$.

- Why then photon is a "true" particle and phonon a "quasiparticle"?

- The difference is in the nature of lattice **vertices**!

- For phonons, the vertices are particles themselves - atoms.
- Phonons emerge from atoms (not the other way around), so atoms are **more fundamental** particles than phonons.
- In this sense, a phonon is "less" particle than an atom, so it makes sense to call it "quasiparticle".

- For photons, the "vertices" are simply fields $\mathbf{E}$, $\mathbf{B}$ at points $\mathbf{x}$.
- In the Standard Model (SM) there are no more fundamental particles at field vertices.
- Hence from SM point of view, photon can be considered a fundamental particle, not "quasiparticle".

- But if SM is not fundamental, then photon can be a "quasiparticle".

Emergence of QFT and Lorentz invariance:

- It is usually considered that relativistic QFT is fundamental,
while non-relativistic QM is only an approximation.
- I propose that (in the more fundamental theory)
the opposite is the case. How could that be?

Again, the key is analogy with sound.

Sound satisfies the wave equation

$$\frac{1}{c_s^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi = 0$$

- Lorentz invariant (with speed of sound c_s instead of c).
- Valid only at distances much larger than interatomic distances.
- Derived from non-relativistic motion of atoms.
- Atoms make the "ether" for sound waves.
- If one observed **only** the sound and nothing else,
it would look as if there was no "ether" for sound.

- First quantization of $\psi \Rightarrow$ QM of a single phonon.
- Second quantization of $\psi \Rightarrow$ QFT of phonons.
- Standard tools in condensed matter.
- Derived from non-relativistic QM of atoms (nuclei + electrons).
- Creation/destruction of phonons from fixed number of atoms.

- By analogy, all relativistic "elementary particles" of Standard Model (photons, electrons, ...) might be derivable from hypothetic more fundamental non-relativistic particles.
- The world looks "fundamentally" relativistic only because we don't yet see those more fundamental degrees.

- It's a neo-Lorentzian ether theory.
- Michelson-Morley experiment ruled out a possibility that Earth moves **through** ether.
- No experiment ruled out a possibility that Earth (and everything else) is **made of** ether.

Example: A Phonon and its Bohmian interpretation:

- Crystal lattice made of N atoms with positions

$$\vec{q} = (\mathbf{q}_1, \dots, \mathbf{q}_N)$$

- Wave function $\Psi(\vec{q}, t)$ satisfies non-relativistic Schrödinger equation

$$\left[\sum_{a=1}^{N} \frac{\hat{\mathbf{p}}_a^2}{2m_a} + V(\vec{q})\right] \Psi = i\hbar\partial_t \Psi$$

- Let $\Psi_{\mathbf{p}}(\vec{q}, t)$ = solution corresponding to 1 (acoustic) phonon with momentum $\mathbf{p}$.

$\Rightarrow$ Most general 1-phonon solution

$$\Psi(\vec{q}, t) = \sum_{\mathbf{p}} c_{\mathbf{p}} \Psi_{\mathbf{p}}(\vec{q}, t).$$

- In the abstract Hilbert space this state is

$$|\Psi(t)\rangle = \sum_{\mathbf{p}} c_{\mathbf{p}} |\Psi_{\mathbf{p}}(t)\rangle$$

which can also be represented by a 1-quasiparticle wave function

$$\psi(\mathbf{x}, t) = \sum_{\mathbf{p}} c_{\mathbf{p}} e^{-i[\omega(\mathbf{p})t - \mathbf{p}\cdot\mathbf{x}]}$$

- units $\hbar = 1$, $\omega(\mathbf{p}) = c_s|\mathbf{p}|$ - Lorentz invariant dispersion relation

- The 1-quasiparticle wave function $\psi(\mathbf{x},t)$ satisfies wave equation

$$\frac{1}{c_s^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi = 0$$

Bohmian interpretation 1:
$\psi(\mathbf{x},t)$ suggests phonon position $\mathbf{X}(t)$.
- Makes sense if one imagines that phonon is fundamental.

Bohmian interpretation 2:
- Denies $\mathbf{X}(t)$, but $\Psi(\vec{q},t)$ suggests atom positions $\vec{Q}(t)$.
- Makes sense if one imagines that atoms are fundamental.

Contains 3 wave-like objects:
1) 1-phonon wave function $\psi(\mathbf{x},t)$, relativistic, not fundamental.
2) Multi-atom wave function $\Psi(\vec{q},t)$, non-relativistic, fundamental.
3) Collective motion of atoms $\vec{Q}(t)$, non-relativistic, fundamental.

Bohmian interpretation 3:

- Denies $\vec{Q}(t)$, but accepts Standard Model fields $\Phi(\mathbf{x}, t)$.
- We discussed it in more detail in Part 2.
- Makes sense if SM is fundamental.

Bohmian interpretation 4:

- Denies $\Phi(\mathbf{x}, t)$, but accepts existence of as yet unknown **truly** fundamental particles with $\vec{Q}_{\text{truly fundamental}}(t)$.
- Truly fundamental particles are not created and destroyed.

$\Rightarrow$ Described by non-relativistic QM.

- Such a more fundamental theory looks natural from a Bohmian point of view.

- So far it's just a general theoretical framework for a research program.
- It's still far from a closed theory.

Possible topics for next talks:

Quantum Foundations VI: Suggestions welcome