Question – Random Processes

Let $X_n = X(n)$, $(n=1,2,\ldots)$ be an independent identically distributed (IID), discrete-time random process, with mean $E[X_n]=a$ (where a is constant) and variance $\mathrm{Var}(X_n)=\sigma_X{}^2$. Let $X_n \sim \mathcal{N}(a,\sigma_X{}^2)$. Now, a new random process S_m is defined as

$$S_m = \sum_{n=1}^{m} X_n$$

Which has a mean $\mu_{S_m} = ma$ and variance $\sigma_{S_m}^2 = m\cdot\sigma_X{}^2$.
Find the common PDF $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n\,;t_1,\ldots,t_n)$ of random variables $X_1,\ldots,X_n$.

Solution:

According to the formal solution, since all the processes are independent, then

$$f_{X_1,\ldots,X_n}(x_1,\ldots,x_n\,;t_1,\ldots,t_n) = \sum_{n=1}^{m} f_{X_n}(x_n\,;t_n) \quad .$$

but I just cannot figure out why exactly a sum is used instead of a product, because of the independence of the processes.
Can someone please indicate what exactly do I miss here?
Thanks.