I have to prove that

$f_n(s) = E(s^{X_n})$ satisfies $f_n(s) = f(f_{n-1}(s))$ $\forall\ n \geq 2$

by conditioning on the value of X_1.

$$f_n(s) = E(s^{X_n}) = \sum_{j=0}^{\infty} E(s^{X_n} | X_1 = j) \cdot P(X_1 = j)$$

My difficulty is to find $E(s^{X_n} | X_1 = j)$

How do I rewrite X_n, by conditioning on X_1?

for example, $X_n | X_{n-1} = j = \sum_{i=1} Z_i$