I need to calculate:

$$M(j,\bar{R}) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\bar{R}}^{+\bar{R}} z^j e^{-\frac{z^2}{2}} dz; \tag{1}$$

Here is how I proceeds:

$$M(2j,\bar{R}) = (2j-1)M(2j-2,\bar{R}) - \frac{2}{\sqrt{2\pi}} \bar{R}^{\frac{2j-1}{2}} e^{-\frac{\bar{R}}{2}} \tag{2}$$

$$= (2j-1)!!(M(0,\bar{R}) - \frac{2}{\sqrt{2\pi}} e^{-\frac{\bar{R}}{2}} \sum_{k=1}^{j} \frac{1}{(2k-1)!!} \bar{R}^{\frac{2k-1}{2}}); \tag{3}$$

$$X \equiv 2e^{-\frac{\bar{R}}{2}} \sum_{k=1}^{j} \frac{1}{(2k-1)!!} \bar{R}^{\frac{2k-1}{2}} : \quad \frac{dX}{d\bar{R}} = e^{-\frac{\bar{R}}{2}} \bar{R}^{-\frac{1}{2}} (1 - \frac{1}{(2j-1)!!} \bar{R}^{\frac{2j}{2}}); \tag{4}$$

$$z \equiv \sqrt{\bar{R}} : \quad X = \int \frac{dX}{d\bar{R}} d\bar{R} = 2 \int_0^{\sqrt{\bar{R}}} e^{-\frac{z^2}{2}} (1 - \frac{1}{(2j-1)!!} z^{2j}) dz; \tag{5}$$

$$M(2j,\bar{R}) = (2j-1)!!(M(0,\bar{R}) - M(0,\sqrt{\bar{R}})) + M(2j,\sqrt{\bar{R}}); \tag{6}$$

$$M(2j,\bar{R}) - (2j-1)!!M(0,\bar{R}) = M(2j,\sqrt{\bar{R}}) - (2j-1)!!M(0,\sqrt{\bar{R}}); \tag{7}$$

The last formula should no longer depends on $\bar{R}$. Because when $\bar{R}$ approaches ∞, $M(2j,\bar{R})$ approaches $(2j-1)!!$ while $M(0,\bar{R})$ approaches 1, the conclusion of the above formula is:

$$M(2j,\bar{R}) = (2j-1)!!M(0,\bar{R}); \tag{8}$$

Unfortunately, this conclusion is empirically wrong.

Where have I made an mistake?