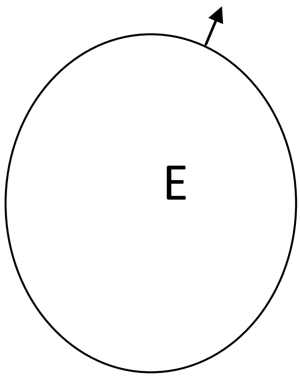
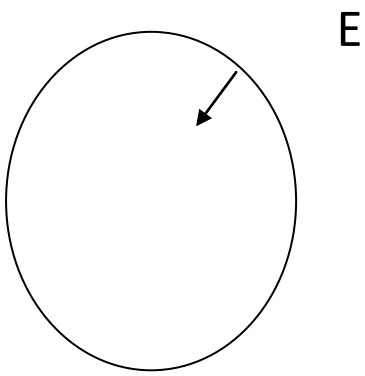
If we have some sort of shape, we choose our normal by the Field presented to us.

Usually when the Field is inside the surface we choose the surface vector to be from the surface, out.

In that case, for the next shape, we choose the following surface vector:

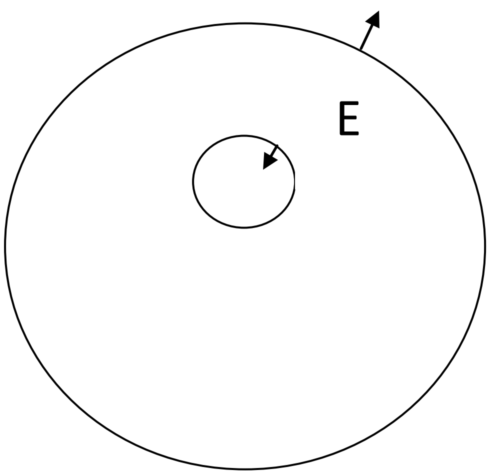


But if the Field is on the outside of the surface we choose it to be "towards r=0"



If you want, flux going in or out.

In my problem I have the next spherical shape with Electric Field E between the spheres, so if I understand, the surface vectors should be in the following directions:



We'll sign the outer sphere as S, and the inner S'.

If we get this eq. (the reason doesn't really matter but if you want to know it's about solving helmoltz eq. using Green)

$$(1)\int_{S+S'}\left(u\frac{\partial v}{\partial n}-v\frac{\partial u}{\partial n}\right)ds=0$$

Where we define:

$$(2)\frac{\partial f}{\partial n}=grad(f)\cdot ds$$

If f=f(r)

That means that in the outer surface S $\left(ds=\hat{r}\right)$

$$(3)\frac{\partial f}{\partial n}=\frac{\partial f}{\partial r}$$

Because the surface vector is going out

In the inner surface S' it should be $\left(ds=-\hat{r}\right)$

$$(4)\frac{\partial f}{\partial n}=-\frac{\partial f}{\partial r}$$

Now, if we know the Electric field an it's derivative on S and letting the function u,v be the next functions:

$$(5)u=E;\quad v=\frac{e^{j\rho r}}{\rho}$$

We get

$$(6)\underbrace{\int_S\left[E\frac{\partial}{\partial n}\left(\frac{e^{j\rho r}}{\rho}\right)-\frac{e^{j\rho r}}{\rho}\frac{\partial E}{\partial n}\right]ds}_{Known}=-\int_{S'}\left[-E\left(\frac{jk}{\rho}-\frac{1}{\rho^2}\right)e^{jk\rho}-\frac{1}{\rho}e^{jk\rho}\frac{\partial E}{\partial n}\right]ds$$

And finally if let the inner radius limit to 0 I'll get

$$(7)E(0)=-\frac{1}{4\pi}\int_S\left[E\frac{\partial}{\partial n}\left(\frac{e^{jkr}}{r}\right)-\frac{e^{jkr}}{r}\frac{\partial E}{\partial n}\right]ds$$

In class we got

$$(8)\underbrace{\int_S \left[E\frac{\partial}{\partial n}\left(\frac{e^{j\rho r}}{\rho}\right) - \frac{e^{j\rho r}}{\rho}\frac{\partial E}{\partial n}\right] ds}_{Known} = -\int_{S'} \left[E\left(\frac{jk}{\rho} - \frac{1}{\rho^2}\right)e^{jk\rho} - \frac{1}{\rho}e^{jk\rho}\frac{\partial E}{\partial n}\right] ds$$

$$(9) E(0) = \frac{1}{4\pi}\int_S \left[E\frac{\partial}{\partial n}\left(\frac{e^{jkr}}{r}\right) - \frac{e^{jkr}}{r}\frac{\partial E}{\partial n}\right] ds$$

So, who has the mistake? Me or my professor?