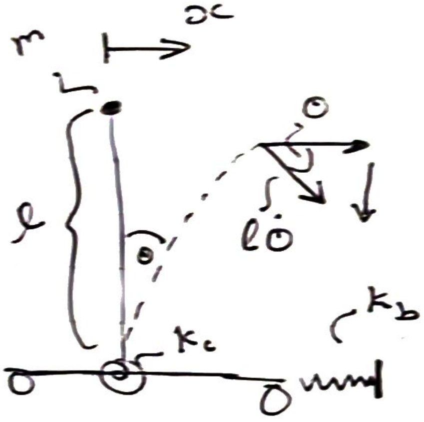
02/12/2024



vertical ⇒ $\sin\theta = \frac{y}{l\dot\theta}$ $y = l\dot\theta\sin\theta$

horizontal ⇒ $x = l\dot\theta\cos\theta$

Total velocity of system (beam/column)

$$V = \dot x\,\hat I + l\dot\theta\cos\theta\,\hat I + l\dot\theta\sin\theta\,\hat J$$

$$V^2 = (\dot x + l\dot\theta\cos\theta)^2 + (l\dot\theta\sin\theta)^2$$

$$= (\dot x + l\dot\theta\cos\theta)(\dot x + l\dot\theta\cos\theta) + (l\dot\theta\sin\theta)^2$$

$$= \dot x^2 + 2\dot x\dot\theta l\cos\theta + (l\dot\theta\cos\theta)^2 + (l\dot\theta\sin\theta)^2$$

$$= \dot x^2 + 2\dot x\dot\theta l\cos\theta + l^2\dot\theta^2$$

$$T = \frac{1}{2}mv^2 + \frac{1}{2}I\dot\theta^2 \qquad U = \frac{1}{2}k_c\theta^2 + \frac{1}{2}k_b x^2 + mgl(1-\cos\theta)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot q}\right) - \frac{\partial T}{\partial q} - \frac{d}{dt}\left(\frac{\partial U}{\partial \dot q}\right) + \frac{\partial U}{\partial q} + \frac{\partial D}{\partial \dot q} = 0 \quad \text{---} \quad (1)$$

$\underline{\underline{x}}$

$$\frac{\partial T}{\partial \dot x} = \frac{m}{2}(2\dot x + 2\dot\theta l\cos\theta) \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot x}\right) = \frac{m}{2}(2\ddot x + 2\ddot\theta l\cos\theta - 2\dot\theta^2 l\sin\theta)$$

$$= m(\ddot x + \ddot\theta l\cos\theta - \dot\theta^2 l\sin\theta)$$

$$\frac{\partial U}{\partial \dot x} = 0 \quad \frac{\partial T}{\partial x} = 0 \qquad \frac{\partial D}{\partial \dot x} = \underline{\underline{c_1\dot x}}$$

$$\frac{\partial U}{\partial x} = k_b x$$

$$m(\ddot x + \ddot\theta l\cos\theta - \dot\theta^2 l\sin\theta) + k_b x + c_1\dot x = 0$$

$$m\ddot x + m(\ddot\theta l\cos\theta - \dot\theta^2 l\sin\theta) + k_b x + c_1\dot x = 0$$

$$m\ddot x = -m(\ddot\theta l\cos\theta - \dot\theta^2 l\sin\theta) - k_b x - c_1\dot x$$

$$\ddot x = -(\theta l\cos\theta - \dot\theta^2 l\sin\theta) + \frac{-k_b x - c_1\dot x}{m}$$

$\underline{\underline{\theta}}$

$$\frac{\partial T}{\partial \dot{\theta}} = \frac{1}{2} m \left(2\dot{x} l \cos\theta + 2 l^2 \dot{\theta}\right) + I \dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \frac{m}{2}\left(2\ddot{x} l \cos\theta - 2\dot{x}\dot{\theta}\sin\theta + 2 l^2 \ddot{\theta}\right) + I\ddot{\theta}$$

$$= m\left(x l \cos\theta - \dot{x}\dot{\theta}\sin\theta + l^2\ddot{\theta}\right) + I\ddot{\theta}$$

$$\frac{\partial T}{\partial \theta} = \frac{1}{2} m \left(-2\dot{x}\dot{\theta} l \sin\theta\right) = -m\dot{x}\dot{\theta} l \sin\theta$$

$$\frac{\partial U}{\partial \dot{\theta}} = 0 \qquad \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{\theta}}\right) = 0$$

$$\frac{\partial U}{\partial \theta} = k_c\theta + (+mgl\sin\theta) = k_c\theta + mgl\sin\theta$$

$$m\left(\ddot{x} l\cos\theta - \cancel{l}\dot{x}\dot{\theta}\sin\theta + l^2\ddot{\theta}\right) + I\ddot{\theta} + m\dot{x}\dot{\theta} l\sin\theta + k_c\theta + mgl\sin\theta + C_2\dot{\theta} = 0$$

$$l^2 m\ddot{\theta} + I\ddot{\theta} + m\left(\ddot{x} l\cos\theta - \cancel{l}\dot{x}\dot{\theta}\sin\theta\right) + m\dot{x}\dot{\theta} l\sin\theta + k_c\theta + mgl\sin\theta + C_2\dot{\theta} = 0$$

$$\ddot{\theta}(l^2 m + I) = -m\left(\ddot{x} l\cos\theta - \cancel{l}\dot{x}\dot{\theta}\sin\theta\right) - m\dot{x}\dot{\theta} l\sin\theta - k_c\theta - mgl\sin\theta - C_2\dot{\theta}$$

$$= -m\left(\ddot{x} l\cos\theta - \dot{x}\dot{\theta} l\sin\theta\right) - m\dot{x}\dot{\theta} l\sin\theta - k_c\theta - mgl\sin\theta - C_2\dot{\theta}$$

$$= -m\ddot{x} l\cos\theta \cancel{- 2\dot{x}\dot{\theta} l\sin\theta} - k_c\theta - mgl\sin\theta - C_2\dot{\theta}$$

$$\ddot{\theta} = \frac{-m\ddot{x}\overset{l\cos\theta}{\wedge} - k_c\theta - mgl\sin\theta - C_2\dot{\theta}}{(l^2 m + I)}$$

$$\dot{\theta}^2 l\sin\theta + \frac{-k_b x - c_1\dot{x}}{m} = A$$

$$\ddot{x} = \dot{\theta}^2 l\sin\theta - \ddot{\theta} l\cos\theta + \frac{-k_b x - c_1\dot{x}}{m} \quad \text{--- } 1$$

$$\ddot{\theta} = \frac{-m\ddot{x}l\cos\theta \overbrace{- k_c\theta - mgl\sin\theta - c_2\dot{\theta}}^{B}}{l^2 m + I} \quad \text{--- } 2$$

sub 1 in 2

$$\ddot{\theta} = \frac{-m(-\ddot{\theta}l\cos\theta + A)^{l\cos\theta} + B}{l^2 m + I} = \frac{-m(-\ddot{\theta}l^2\cos^2\theta + Al\cos\theta) + B}{l^2 m + I}$$

$$= \frac{m\ddot{\theta}l^2\cos^2\theta}{l^2 m + I} + \frac{-m(Al\cos\theta) + B}{l^2 m + I}$$

RHS: $\dfrac{\ddot{\theta} - m\ddot{\theta}l^2\cos^2\theta}{l^2 m + I}$

$$\ddot{\theta}\cdot\underbrace{\left[1 - \frac{ml^2\cos^2\theta}{l^2 m + I}\right]}_{C}$$

$$\ddot{\theta}C = \frac{-m(Al\cos\theta) + B}{l^2 m + I} \qquad \ddot{\theta} = \left[\frac{-m(Al\cos\theta) + B}{l^2 m + I}\right]\frac{1}{C}$$