Help Me Understand Relativity

Approximatelysphere

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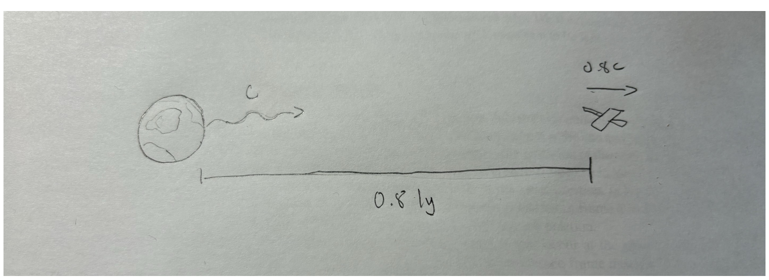
1 The problem

I'm having trouble with this specific problem/though experiment: Suppose you have a satellite travelling at $v = 0.8c$ in the $+x$ direction and at $t = 0$ it aligns with the earth. At $t = 1$ year in the earth's frame of reference, a signal from mission control on earth is sent to the satellite. How long does it take for the signal to travel from earth to the satellite, in the earth's frame and in the satellite's frame?

2 My solution and confusion

At $t = 1$ year, the satellite would have travelled 0.8ly according to the earth's frame. So if a signal is sent, the total time would include the time needed for light to travel 0.8ly plus the time needed for light to catch up to the satellite travelling at 0.8c.



Denote Δt as time needed in earth's frame, and $\Delta t'$ to be the time needed in the satellite's frame. Hence:

$$c\Delta t = 0.8\text{ly} + 0.8c\Delta t$$

$$0.2c\Delta t = 0.8\text{ly}$$

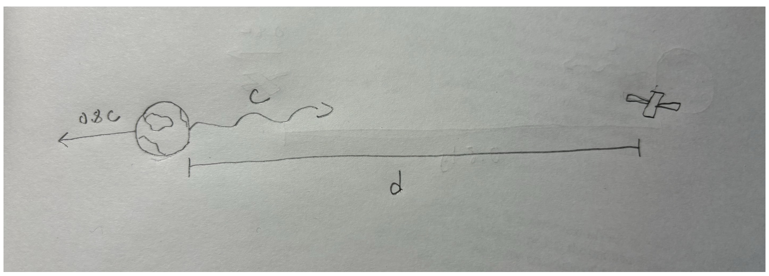
$$\Delta t = 4 \text{ years}$$

We can then use the time dilation formula, $\Delta t = \gamma \Delta t' = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$, to figure out the time passed in the satellite's frame:

$$\Delta t' = \Delta t \cdot \sqrt{1 - \frac{(0.8c)^2}{c^2}} = 4 \cdot \sqrt{0.36} = 4 \cdot 0.6 = 2.4$$

So, by this method, we find that 4 years have passed in the earth's frame and 2.4 years have passed in the satellite's frame.

However, this is where my confusion starts. If we now consider events from the satellite's frame of reference, the satellite is stationary while the earth is moving at $0.8c$ in the $-x$ direction as shown:



Since c is constant in all frames of reference, it still travels at c here even though the earth is moving in the opposite direction. Since the earth frame is moving, it should experience length contraction when observed from the satellite's frame. We can find this contracted length, d, from the length contraction formula: $d = \sqrt{1 - \frac{v'^2}{c^2}} L$, where $v' = -0.8c$ is the speed at which the earth's frame is moving with respect to the satellite and $L = 0.8\text{ly}$ is the "proper length" of the separation between the earth and the satellite in the earth's reference frame. Hence:

$$\begin{aligned} d = \sqrt{1 - \frac{v'^2}{c^2}} L &= \sqrt{1-(-0.8)^2} \cdot 0.8 \\ &= \sqrt{0.36} \cdot 0.8 \\ &= 0.6 \cdot 0.8 \\ &= 0.48\text{ly} \end{aligned}$$

The time, $\Delta t'$ needed is simply d over the speed of light. Therefore, we have $\Delta t' = \frac{d}{c} = \frac{0.48\text{ly}}{c} = 0.48$ years. Clearly, $0.48 \neq 2.4$, so something is off. What is wrong with my reasoning?