

$$k_n \sim \frac{n\pi}{a_0 \chi_s} \quad (7.81)$$

Using (7.74) we expect a series of peaks in the spectrum at the associated multipoles, $\ell \sim n\ell_s \sim 300n$. As explained below, because of the non-negligible radiation density at recombination, the position of the peaks are shifted to lower multipoles. However, the spacing of the peaks is maintained to good approximation and one observes

$$\ell_n \sim n\ell_s - \Delta\ell \quad (\ell_s, \Delta\ell) \sim (300, 80), \quad n = 1, 2, 3, \dots \quad (7.82)$$

where ℓ_s is given by (7.75). As seen in Fig. 7.11, at least five peaks are apparent in the anisotropy spectrum.

We will see in Sect. 7.5 that the large peculiar velocities of the plasma for $\Theta_{\text{rec}} = (n + 0.5)\pi$, $n = 1, 2, 3, \dots$ generate peaks in the spectrum of the CMB polarization. These peaks, shown in Fig. 7.17, have been seen by the QUAD collaboration [157].

The shift $\Delta\ell$ in (7.82) depends on the ratio of the radiation and matter densities at recombination because of the “early-time integrated Sachs–Wolf (ISW) effect” [154]. This effect is due to the decay of the gravitational potential between t_{enter} and t_{eq} . The potential perturbation for a mode of wavelength λ is proportional to $\rho(\Delta\rho/\rho)\lambda^2$. During the matter epoch, the potential is constant because $\rho \propto a^{-3}$, $\Delta\rho/\rho \propto a$, and $\lambda^2 \propto a^2$. During the radiation epoch, perturbations inside the Hubble radius oscillate ($\Delta\rho/\rho$ constant) so the potential decays. Potential decay occurs mostly during the first compression. It is timed so as to give a kick to the mode, increasing its amplitude and decreasing the time to reach the first compression. This has the effect of pushing the first peak to larger angular scales. An empirical formula is [156]:

$$\Delta\ell \sim \ell_s \times 0.267 \left(\frac{\rho_R(a_{\text{rec}})}{\rho_M(a_{\text{rec}})} \right)^{0.1} \sim 72 \frac{\ell_s}{300} \left(\frac{0.27}{\Omega_M h_{70}^2} \right)^{0.1} \quad (7.83)$$

Besides the series of peaks for $\ell > 200$, the spectrum in Fig. 7.11 exhibits two interesting effects, the late time integrated Sachs–Wolfe effect at $\ell < 5$ and damping at $\ell > 1000$. Like the early-time ISW effect, its late-time analog comes about through potential decay, this time due to the onset of vacuum energy domination. A photon that enters a potential fluctuation will leave the fluctuation later when the depth is smaller. This causes the photon energy to be boosted by the change in the potential between entry and exit. This adds temperature fluctuation at large scale (because the change in potential is proportional to the time spent in the potential). It is responsible for the rise in the C_ℓ for $\ell < 5$ seen in Fig. 7.11. It also generates a correlation between temperature and the number density of foreground galaxies. The correlation with SDSS galaxies has been seen [155]. All these measurements confirm that $\Omega_M < 1$ providing independent evidence for dark energy.

The rapid decrease of the C_ℓ for $\ell > 1000$ is due to the fact that recombination is not an instantaneous event but takes place over a non-zero time interval. Photons