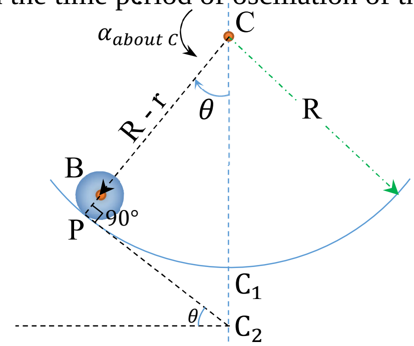
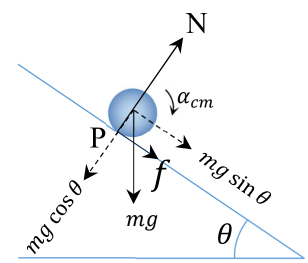
Question: A solid spherical ball of mass *m* and radius *r* rolls without slipping on a rough concave surface of large radius *R*. Find the magnitude and direction of friction on the ball, and the time period of oscillation of the ball.



Solution: Let at any point of time, the ball be at angle theta from the line joining the point C (centre of curvature of the concave surface) to point C_1.

Since we have to calculate the time period of the ball, and the ball is undergoing angular simple harmonic motion about an axis passing through the point C and perpendicular to the paper, therefore, we will calculate the torque about point C and then find angular acceleration, α about point C and that will give us the angular frequency ω about point C, and thereby time period.

Let B be the centre of the ball. P is the point of contact of ball with the surface. Line PC_2 is tangent to the surface at the point of contact P.



The forces acting on the ball is shown in the diagram beside (→). Since the surface is rough, there must be friction acting at the point of contact of the ball and the surface. Let the direction of friction be as shown in the pic. Since we have to find the angular acceleration about point C, therefore, we need to calculate torque about point C. The forces acting on the ball are gravity(mg), Normal reaction (N) and friction.

Out of these 3 forces, only friction and mgsinθ will create a torque about point C. So, now we need to calculate the frictional force acting on the ball.

Since the ball rolls without slipping, i.e. in pure rolling motion, therefore, the acceleration of point of the ball which is instantaneously in contact with the surface, must be zero.

i.e. $a_{poc} = 0$

for the ball, $a_{cm} = \frac{mgsin\theta + f}{m}$. Let α_{cm} be the angular acceleration of the ball, about the ball's centre of mass. $\Rightarrow a_{poc} = a_{cm} - \alpha_{cm} r.$

Now, torque about the centre of mass of the ball, $\tau_{cm} = -fr$. Minus sign has been taken here since the direction of torque is opposite to the shown direction of α_{cm}.

Moment of Inertia of the ball about the axis passing through it's centre, $I_{cm} = \frac{2}{5}mr^2$. Now, $\tau_{cm} = I_{cm}\alpha_{cm}$

$\Rightarrow -fr = \frac{2}{5}mr^2\alpha_{cm} \Rightarrow \alpha_{cm} = -\frac{5}{2}\frac{fr}{mr^2} = -\frac{5}{2}\frac{f}{mr}$

$\therefore\ a_{poc} = a_{cm} - \alpha_{cm} r = \frac{mgsin\theta + f}{m} + \frac{5}{2}\frac{f}{m} = g\sin\theta + \frac{7}{2}\frac{f}{m}$, and since $a_{poc} = 0$

$\Rightarrow g\sin\theta + \frac{7}{2}\frac{f}{m} = 0 \Rightarrow \frac{7}{2}\frac{f}{m} = -g\sin\theta \Rightarrow f = -\frac{2}{7}mg\sin\theta$. The minus sign here means that the direction of friction is up the incline.