Brief illustration B.4 The trajectory of a particle

Consider an argon atom free to move in one direction (along the x-axis) in a region where $V=0$ (so the energy is independent of position). Because $v=\mathrm{d}x/\mathrm{d}t$, it follows from eqns B.1 and B.8 that $\mathrm{d}x/\mathrm{d}t=(2E_\mathrm{k}/m)^{1/2}$. As may be verified by substitution, a solution of this differential equation is

$$x(t)=x(0)+\left(\frac{2E_\mathrm{k}}{m}\right)^{1/2}t$$

The linear momentum is

$$p(t)=mv(t)=m\frac{\mathrm{d}x}{\mathrm{d}t}=(2mE_\mathrm{k})^{1/2}$$

and is a constant. Hence, if we know the initial position and momentum, we can predict all later positions and momenta exactly.

Self-test B.4 Consider an atom of mass m moving along the x direction with an initial position x_1 and initial speed v_1. If the atom moves for a time interval Δt in a region where the potential energy varies as $V(x)$, what is its speed v_2 at position x_2?

Answer: $v_2=v_1-\left|\mathrm{d}V(x)/\mathrm{d}x\right|_{x_1}\Delta t/m$