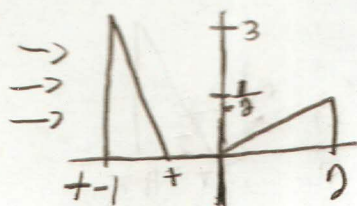


c) Compute the convolution $y(t) = x(t) * h(t)$. Your answer should be in closed-form (e.g., no integrals).

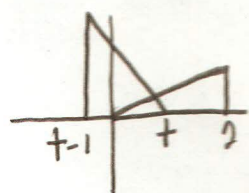


$$\begin{aligned} h(t-\tau) &= 3(t-\tau) \\ h(t) &= 3t \\ h(\tau) &= 3\tau \\ h(t-\tau) &= 3(t-\tau) \\ &= 3t - 3\tau \end{aligned}$$



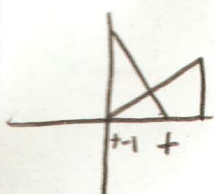
$$t > 0$$

$$y(t) = 0$$



$$\begin{aligned} t = 0 \\ t - 1 = 0 \Rightarrow 0 < t < 1 \\ t = 1 \end{aligned}$$

$$\begin{aligned} y(t) &= \int_0^t \frac{1}{2} + (3t - 3\tau) d\tau = \frac{3}{2}t + \int_0^t d\tau - \frac{3}{2}t \int_0^t \tau d\tau \\ &= \frac{3}{2}t^2 [\tau]_0^t - \frac{3}{2}t [\frac{\tau^2}{2}]_0^t = \frac{3}{2}t^3 - \frac{3}{4}t^3 = \frac{3}{4}t^3 \end{aligned}$$

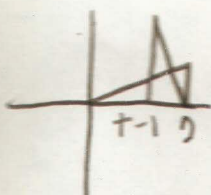


$$\begin{aligned} t = 1 \\ t - 1 = 1 \\ t = 2 \end{aligned} \quad 1 < t < 2$$

$$y(t) = \int_{t-1}^t \frac{1}{2} + (3t - 3\tau) d\tau$$

$$\begin{aligned} &= \frac{3}{2}t^2 [\tau]_{t-1}^t - \frac{3}{4}t [\tau^2]_{t-1}^t = \frac{3}{2}t^2 [t - (t-1)] - \frac{3}{4}t [t^2 - (t-1)^2] \\ &= \frac{3}{2}t^2 - \frac{3}{4}t (2t - 1) = \frac{3}{2}t^2 - \frac{6}{4}t^2 + \frac{3}{4}t = \frac{3}{4}t \end{aligned}$$

$\begin{aligned} &t^2 - (t^2 - 2t + 1) \\ &t^2 - t^2 + 2t - 1 \\ &2t - 1 \end{aligned}$



$$\begin{aligned} t = 2 \\ t - 1 = 2 \\ t = 3 \end{aligned} \quad 2 < t < 3$$

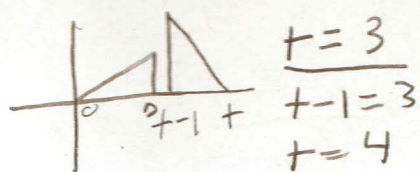
$$y(t) = \int_{t-1}^2 \frac{1}{2} + (3t - 3\tau) d\tau = \frac{3}{2}t^2 [\tau]_{t-1}^2 - \frac{3}{4}t [\tau^2]_{t-1}^2$$

$$\begin{aligned} &= \frac{3}{2}t^2 [2 - (t-1)] - \frac{3}{4}t [2^2 - (t-1)^2] = \frac{9}{2}t^2 - \frac{3}{2}t^3 + \frac{3}{4}t^3 - \frac{6}{4}t^2 + \frac{9}{4}t \\ &\quad \begin{aligned} &3 - t \\ &4 - (t^2 - 2t + 1) = -t^2 + 2t + 3 \end{aligned} \end{aligned}$$

3c) Continued

for $t=2$ we got

$$\begin{aligned} y(t) &= \frac{9}{2}t^2 - \frac{3}{2}t^3 + \frac{3}{4}t^3 - \frac{6}{4}t^2 - \frac{9}{4}t \\ &= \frac{18}{4}t^2 - \frac{6}{4}t^2 - \frac{6}{4}t^3 + \frac{3}{4}t^3 - \frac{9}{4}t \\ &= \frac{12}{4}t^2 - \frac{3}{4}t^3 - \frac{9}{4}t = -\frac{3}{4}t^3 + 3t^2 - \frac{9}{4}t \end{aligned}$$



$t \geq 3$

$$y(t) = 0$$

$$y(t) = \begin{cases} 0, & t < 0 \\ \frac{3}{4}t^3, & 0 \leq t < 1 \\ \frac{3}{4}t, & 1 \leq t < 2 \\ \frac{-3 + (t^2 - 4t + 3)}{4}, & 2 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$