

\* Projective measurement of a qubit.

①

Ex :  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Sol : measurements operators :

$$M_0 = |0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} ; M_1 = |1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

measurement probabilities :

$$P(0) = \langle \psi | M_0^\dagger M_0 | \psi \rangle = \alpha^* \alpha \langle 0 | 0 \rangle = |\alpha|^2$$

$$P(1) = \langle \psi | M_1^\dagger M_1 | \psi \rangle = \beta^* \beta \langle 1 | 1 \rangle = |\beta|^2$$

State after measurement :

$$\frac{M_0 |\psi\rangle}{\sqrt{P(0)}} = \frac{\alpha |0\rangle}{\sqrt{|\alpha|^2}} = \frac{\alpha}{|\alpha|} |0\rangle$$

$$\frac{M_1 |\psi\rangle}{\sqrt{P(1)}} = \frac{\beta |1\rangle}{\sqrt{|\beta|^2}} = \frac{\beta}{|\beta|} |1\rangle$$