$$\frac{\partial G}{\partial z} + \frac{n_{co}}{c\cos\theta_z}\frac{\partial G}{\partial t} + \alpha G = \frac{3\alpha}{4}\int_0^{\theta_c} G(\theta'_z, z, t)\theta'_z\,d\theta'_z, \tag{5–79}$$

where $\alpha = N\sigma_{tot}$. Given the initial form of G at $z = 0$, we can solve this equation by iteration for small values of α and θ_z.

On-axis beam

We assume that at $z = 0$ the pulse is composed of a uniform collimated beam of on-axis rays, with unit total power. Using Eqs. (5–69) and (5–78), we deduce that

$$G(\theta_z, 0, t) = \delta(t)\delta(\theta_z)/\theta_z; \qquad P_{br}(0) = 1, \tag{5–80}$$

where δ is the Dirac delta function. The lowest order solution of Eq. (5–79) corresponds to $\alpha = 0$. Hence

$$G(\theta_z, z, t) = \delta(t - zn_{co}/c)\,\delta(\theta_z)/\theta_z. \tag{5–81}$$

In other words, when scattering is ignored, the pulse propagates unattentuated and undistorted along the fiber with speed c/n_{co}. To first order, we can consider the scattering effects due to the two terms in α in Eq. (5–79) separately. The term αG on the left of the equation gives rise to an overall attenuation of G by the factor $\exp(-\alpha z)$, while the term on the right accounts for dispersion. The contribution from the latter is found by substituting for G from Eq. (5–81), multiplying by $\exp(-\alpha z)$, and integrating with respect to z with $t = n_{co}z/c\cos\theta_z$. Putting the two contributions together we obtain correct to first order

$$G(\theta_z, z, t) = \left\{\delta\left(t - \frac{zn_{co}}{c}\right)\frac{\delta(\theta_z)}{\theta_z} + \frac{3\alpha}{4}\frac{c}{n_{co}}\frac{\cos\theta_z}{1-\cos\theta_z}\right\}\exp(-\alpha z), \tag{5–82}$$

for $(zn_{co}/c) < t < (zn_{co}/c\cos\theta_z)$, and $G = 0$ otherwise. The first term within the curly brackets represents the undistorted pulse and the second term the distribution of scattered power, while the overall attenuation is due to scattering into leaky rays. Secondary scattering from these rays back into bound rays is ignored in this description.

Attenuation of pulse power

The impulse response $Q(z,t)$ is the integral of G over all ray directions. Noting the limitations on Eq. (5–82), we deduce that for $(zn_{co}/c) < t < (zn_{co}/c\cos\theta_z)$ the impulse