SECTION 2.5 THE VIRTUAL STATISTICAL ENSEMBLE

This is the basic term in the operational interpretation of QM. In this operational basis we must determine on empirical level the corresponding terms for the mathematical/conceptuals previously presented in the axioms of QM. For this purpose, we'll need the concept of "virtual statistical ensemble (vse in the sequel)" put forth by Albert Einstein when he sought an operational basis for classical statistical mechanics.

The first element of a vse is a very big number N of identical quantum systems, all prepared in the same quantum state. Let's consider a relevant observable of the quantum system **A** described according the axioms by a self-adjoint operator A with known spectrum σ (A) – generally a mixed spectrum. Let $a_n \in \sigma$ (A). The vse allows one to find the probability this certain a_n on the ensemble of systems from an operational point of view. Now let $N(a_n)$ be the number of systems from this ensemble for which one obtains a_n upon measuring **A**. The operational probability for this particular value is well approximated by the number

$$P_{operational}(a_n) \propto \frac{N(a_n)}{N}$$

The exact value of this operational probability is obtained in the virtual limit of indefinite increase of N (number of identically prepared systems from the ensemble)

$$P_{operational}(a_n) = \lim_{N \to \infty} \frac{N(a_n)}{N}$$

In real life one won't need to take the limit for N, just ensure that N is big enough and the central limit theorem from mathematical statistics will enable this conclusion.

Therefore, the operational probability is the probability on the (virtual) statistical ensemble. The theoretical probability postulated in the chapter on the QM axioms (the so-called Born rule) is consistent with the operational probability iff

$$P_{operational}(a_n) = P_{theoretical}(a_n)$$