

The static electric field conserves energy, so the potential energy of a charge distribution is equal to the work required to assemble the charge distribution. The potential energy of a single charge q at position \mathbf{r} in an electric potential $\phi(\mathbf{r})$ is given by

$$U = q\phi(\mathbf{r}). \quad (1)$$

In Eq. (1), $\phi(\mathbf{r})$ is the potential due to charge distributions other than the point charge q itself. Thus, electrostatic ‘self-energy’ of a point charge is excluded.

The potential energy of two point charges would be

$$U_2 = q_2\phi_1(\mathbf{r}_2) = \frac{q_2q_1}{r_{21}}, \quad (2)$$

where $\phi_1(\mathbf{r}_2)$ is the potential at the point \mathbf{r}_2 due to the charge q_1 , and $r_{21} = |\mathbf{r}_2 - \mathbf{r}_1|$ is the magnitude of the vector from q_1 to q_2 . As additional charges are added, the work required to bring the i th charge from infinity to its final location is determined by the potential due to the $i-1$ previous charges:

$$\delta U_i = q_i\phi_{i-1}(\mathbf{r}_i) = q_i \sum_{j=1}^{i-1} \frac{q_j}{r_{ij}}. \quad (3)$$

Then, the potential energy for a collection of N point charges is

$$U_N = \sum_{i=2}^N \delta U_i = \sum_{i=2}^N \sum_{j=1}^{i-1} \frac{q_i q_j}{r_{ij}}. \quad (4)$$

This potential energy can be written in somewhat simpler notation as

$$U_N = \sum_{j < i}^N \frac{q_i q_j}{r_{ij}}. \quad (5)$$

The form of the function being summed in Eq. (5) is symmetric with respect to the indices i and j . This means we can interchange i and j in the sum, and then

$$U_N = \sum_{j < i}^N \frac{q_i q_j}{r_{ij}} = \sum_{i < j}^N \frac{q_j q_i}{r_{ji}}. \quad (6)$$

Adding the two sums in Eq. (6) gives a sum over all i and j with $j \neq i$ that is just twice U_N . Then,

$$U_N = \frac{1}{2} \sum_{j \neq i}^N \frac{q_i q_j}{r_{ij}}, \quad (7)$$

with the factor $\frac{1}{2}$ entering because of the double counting in the sum. The sum is over all N charges.

Looking at the starting point of our derivation of electrostatic energy, we see that the subscript, $i \neq j$, in Eq. (7), means that a point charge does not interact with itself in getting the energy of point charges. So, what is sometimes called the ‘self-energy’ of a point charge is not there, and should be excluded from any integral.