★★★*2.81 symbolic equations, calculus and synthesizing new specific equations*

A good model for the acceleration of a car trying to reach top speed in the least amount of time is $a_x = a_0 - kv_x$, where a_0 is the initial acceleration and k is a constant.

a. Find an expression for k in terms of a_0 and the car's top speed v_{max}.

b. Find an expression for the car's velocity as a function of time.

c. A MINI Cooper S has an initial acceleration of 4.0 m/s^2. At maximum acceleration, how long does it take the car to reach 95% of its top speed?

$$a_x = a_0 - kV_x \quad [1]$$

(a) $V_x = V_{max}$ when a_x decreases to zero

$\rightarrow a_x = 0 = a_0 - kV_{max}$

$\rightarrow k = \dfrac{a_0}{V_{max}}$

(b)

$$a_x = a_0 - \frac{a_0}{V_{max}} V_x \quad [2]$$

(b)

[Eq 1] $a_x = a_0 - kv_x$

$$\frac{dv}{dt} = a_0 - \frac{a_0}{v_{max}} v$$

$$\frac{dv}{dt} = kv_{max} - kv$$

$$\frac{dv}{dt} = k(v_{max} - v)$$

$$\frac{dv}{dt} = -k(v - v_{max})$$

let $u = v - v_{max}$

$$\frac{du}{dt} = -ku$$

$$u = Ae^{-kt}$$

$$v - v_{max} = Ae^{-kt}$$

$$0 - v_{max} = Ae^{-k\cdot 0}$$

$$A = -v_{max}$$

$$v = v_{max}(1 - e^{-kt})$$

1st method option

- recognize that growth of v is decaying as $v \longrightarrow v_{max}$
- sol'n will likely be exponential decay
- put diff EQ into a familiar form

Only function u that satisfies $\frac{du}{dt} = -ku$ is $u(t) = Ae^{-kt}$

general sol'n

Then for $v(0) = 0$ at $t = 0$

When starting from rest.

(c) $\dfrac{V_f}{V_{max}} = 1 - e^{-kt} = 1 - {}^{t/\tau}$

$\tau = \dfrac{1}{k}$

$e^{-kt} = 1 - 0.95$

$\ln(e^{-kt}] = \ln(0.05)$

$t = \dfrac{-\ln(0.05)}{k}$

$t \overset{?}{=} \dfrac{-V_{max}}{a_0} \overset{?}{} \ln(0.05)$ ✓

$t = (0.75\,V_{max})$ seconds

$= \boxed{50 \text{ sec}}$

t	% V_{max}
0	0
τ	63%
2τ	86%
3τ	95%

$V_{max} \approx 150$ mph

↓

$67 \frac{m}{s}$