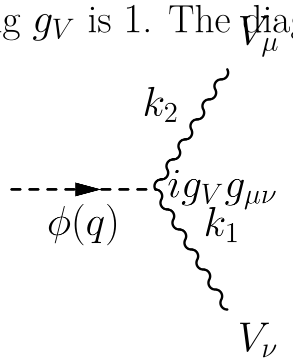
0.0.1 Scalar decay to two gauge bosons

I want to calculate the decay width for the process where gauge bosons (V_μ) are produced from the decay of any scalar field ϕ. The interaction Lagrangian is

$$\mathcal{L}_{\rm int} = g_V\, \phi\, V_\mu V^\mu \tag{1}$$

V_μ is similar to A_μ. Thus, V_μ has mass dimension. Therefore, the mass dimension of coupling g_V is 1. The diagram is as follows:



Feynman amplitude

$$\mathcal{M} = ig_V\, g^{\mu\nu}\, \epsilon^*_\mu(k_1, s_1)\, \epsilon^*_\nu(k_2, s_2) \tag{2}$$

where s_1 and s_2 are the spins of produced particles.

$$|\mathcal{M}|^2 = (g_V m_V)^2 \sum_{s_1} \epsilon^*_\mu(k_1, s_1)\, \epsilon^*_\alpha(k_1, s_1) \sum_{s_2} \epsilon^*_\nu(k_2, s_2)\, \epsilon^*_\beta(k_2, s_2)\, g^{\mu\nu}\, g^{\alpha\beta} \tag{3}$$

Now, we will use the relations $\sum_s \epsilon^*_\mu(k, s)\, \epsilon^*_\alpha(k, s) = -g_{\mu\alpha} + \frac{k_\mu k_\alpha}{m_V^2}$

$$|\overline{\mathcal{M}}|^2 = \frac{\delta_V}{2}(g_V m_V)^2 \left(-g_{\mu\alpha} + \frac{k_{1\mu}k_{1\alpha}}{m_V^2}\right)\left(-g_{\nu\beta} + \frac{k_{2\nu}k_{2\beta}}{m_V^2}\right) g^{\mu\nu} g^{\alpha\beta} \tag{4}$$

$$= \frac{\delta_V}{2}(g_V m_V)^2 \left(g_{\mu\alpha}g_{\nu\beta} - \frac{k_{1\mu}k_{1\alpha}}{m_V^2} g_{\nu\beta} - g_{\mu\alpha}\frac{k_{2\nu}k_{2\beta}}{m_V^2} + \frac{k_{1\mu}k_{1\alpha}}{m_V^2}\frac{k_{2\nu}k_{2\beta}}{m_V^2}\right) g^{\mu\nu} g^{\alpha\beta} \tag{5}$$

$$= \frac{\delta_V}{2}(g_V m_V)^2 \left(4 - \frac{k_1 \cdot k_1}{m_V^2} - \frac{k_2 \cdot k_2}{m_V^2} + \frac{(k_1 \cdot k_2)^2}{m_V^4}\right) \tag{6}$$

$\delta_V/2$ is whether decay is to identical particles or not: $\delta_V = 2$ if not identical particles and $\delta_V = 1$ if identical particles.Here we also have used $g^{\mu\nu}g_{\mu\nu} = 4$.

Now, using $k_1^2 = -m_1^2$, $k_2^2 = -m_2^2$, and $k_1 \cdot k_2 = \left(M_\phi^2 - m_1^2 - m_2^2\right)/2$, where M_ϕ is the mass of ϕ, we get

$$|\overline{\mathcal{M}}|^2 = \frac{\delta_V}{2}(g_V m_V)^2 \left(2 + \frac{\left(M_\phi^2 - m_1^2 - m_2^2\right)^2}{4m_V^4}\right) \tag{7}$$

where $m_1 = m_2 = m_V$ and $\delta_V = 2$, as I choose outgoing particles are identical.