Since the power spectrum has dimensions of volume, its physical meaning is perhaps not immediately obvious. A more easily understood function is the dimensionless function $\Delta(k)$:

$$\Delta^2(k) \equiv \frac{k^3 P(k)}{2\pi^2} \quad \Rightarrow \quad \langle \delta^2(r,t) \rangle = \int_0^\infty \frac{dk}{k} \Delta^2(k) , \qquad (7.31)$$

The function $\Delta^2(k)$ gives the contribution per unit $\ln k$ to the density fluctuations. It is thus a measure of the density fluctuations on the scale k^{-1}.

Figure 7.4 shows the theoretical present-epoch power spectrum $P(k)$ and density fluctuations $\Delta(k)$ for $(\Omega_M, \Omega_\Lambda) = (0.27, 0.73)$ and for $(1, 0)$. The method of calculation will be explained below in Sect. 7.2.1. For small k, $P(k) \propto k$ until reaching a maximum near $k \sim 0.02h\mathrm{Mpc}^{-1}$, for $(0.27, 0.73)$, or $k \sim 0.08h\mathrm{Mpc}^{-1}$, for $(1, 0)$.[1] After the maximum, they fall roughly like $1/k^3$. The density fluctuations $\Delta(k)$ are small at large scale (small k) increasing like $\sim k^2$ before reaching unity at $k \sim 0.1\,\mathrm{Mpc}^{-1}$. This value of k, corresponding to the length scale $k^{-1} \sim 10\,\mathrm{Mpc}$ is the dividing line between the homogeneous universe at large scale and the very clumpy universe at small scale.

Note that $\Delta(k)$ does not decrease with increasing k so the integral (7.31) for the density variance shows no sign of converging. This has no practical importance because we cannot measure the density at individual points. We must always average over regions of non-vanishing volume. In this case, the effective variance of the density function is a function σ_r giving the fluctuations of mass within spheres of radius $rh^{-1}\mathrm{Mpc}$. It is relatively straightforward (Exercise 7.1) to show that

$$\sigma_r^2 = \int_0^\infty \frac{dk}{k} \left[\frac{3 j_1(kr)}{kr} \right]^2 \Delta^2(k) , \qquad (7.32)$$

where $j_1(x) = \sin(x)/x - \cos(x)$. The factor $(3j_1(kr)/kr)^2$ is unity for $kr \ll 1$ and $\rightarrow 0$ for $kr \gg 1$. The mass fluctuation (7.32) is thus the same as the density fluctuation (7.31) except that the integral is cut off for $k > 1/r$.

We saw in Sect. 7.1 that the potential depth of a spherical perturbation in a CDM universe remained constant in time. As such, we can expect that the fluctuation of the Newtonian potential will be very useful in understanding the time evolution of the power spectrum. As with the density, we can expand the potential in plane waves

$$\phi(\mathbf{r}, t) = V^{-1/2} \sum_{\mathbf{k}} \phi_{\mathbf{k}} \exp(i\,\mathbf{k} \cdot \mathbf{r}), \qquad (7.33)$$

The $\phi_{\mathbf{k}}$ can be related to the $\delta_{\mathbf{k}}$ by using the Poisson equation, $\nabla^2\phi = 4\pi G\rho$:

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[1] In discussing the power spectrum, we will follow the convention of the literature by using h rather than h_{70}.