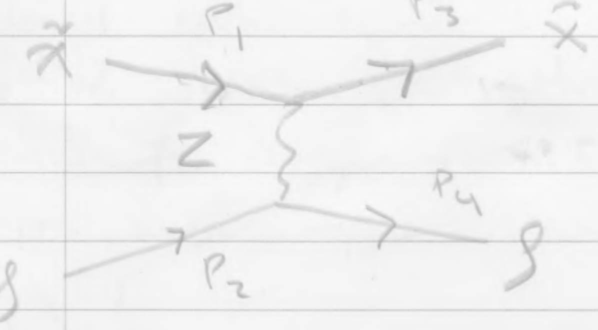
[redacted]

FY3464 Eksamen 11

$\bar{\chi} f \to \bar{\chi} f$ scattering



The Feynman diagram takes into account both scattering and the process.

The next step is then to create a matrix-element. In this we follow the procedure from Thomson: „Modern Particle Physics". With a foundation in figure 1 and the Feynman rules for diagram, we get:

$$i\mathcal{M}_f = \left(\bar{u}(p_3)\left(-g_Z\gamma^\mu \frac{1}{2}\left(c_V^{\tilde{\chi}} - c_A^{\tilde{\chi}}\gamma^5\right)\right) u(p_1)\right)\left(\frac{-i g_{\mu\nu}}{q^2 - m_Z^2}\right)\left(\bar{u}(p_4)\left(-g_Z\gamma^\nu \frac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\right) u(p_2)\right)$$

And this then becomes the expression:

$$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu} \left[\bar{u}(p_3)\gamma^\mu \frac{1}{2}\left(c_V^\nu - c_A^\nu \gamma^5\right) u(p_1) \right] \left[\bar{u}(p_4)\gamma^\nu \frac{1}{2}\left(c_V^f - c_A^f \gamma^5\right) u(p_2) \right]$$

This is the matrix-element. We clearly use the notation from Thomson. And c_V and c_A are notations for V and A currents. V = vector and A = Axial vector as proposed by Feynman and Gell-Mann.

Having found the matrix-element, we will continue on to find the cross section for process (1). The total cross section is given by:

$$\cancel{\sigma = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f^{\,*}|}{|\vec{p}_i^{\,*}|} \int |M_{fi}|^2 \, d\Omega^*}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\left\langle |M_{fi}|^2 \right\rangle$$

which we then integrate over solid angle. But first we must find $\langle |M_{fi}|^2\rangle$.

$$\left(\frac{-g_z^2}{q^2 - m_z^2}\, g_{\mu\nu} \left[\bar{u}(p_3)\, \gamma^\mu \, \frac{1}{2}\left(c_V^{\tilde{x}} - c_A^{\tilde{x}} \gamma^5\right) u(p_1)\right]\left[\bar{u}(p_4)\, \gamma^\nu \,\frac{1}{2}\left(c_V^{f} - c_A^{f}\gamma^5\right) u(p_2)\right]\right) \cdot \left(-\frac{g_z^2}{q^2 - m_z^2}\, g_{\mu\nu}\left[\bar{u}(p_3)\, \gamma_\nu \,\frac{1}{2}\left(c_V^{\tilde{x}} - c_A^{\tilde{x}}\gamma^5\right)\bar{u}(p_1)\right]\left[\bar{u}(p_4)\,\gamma_\mu\,\frac{1}{2}\left(c_V^{f} - c_A^{f}\gamma^5\right)\bar{u}(p_2)\right]^{\dagger}\right)$$

We use trace on this equation and get:

$$\mathrm{Tr}\left(\left(\bar{w}(P_3)\gamma^{\mu}\frac{1}{2}\left(C_V^{\tilde{x}}-C_A^{\tilde{x}}\gamma^5\right)\right.\right.$$

$$\left.u(P_1)\right)\times\mathrm{Tr}\left(\bar{w}(P_4)\gamma^{\nu}\frac{1}{2}\left(C_V^{\rho}-C_A^{\rho}\right.\right.$$

$$\left.\left.\gamma^5\right)u(P_2)\right)\times\mathrm{Tr}\left(u(P_3)\gamma_{\nu}\cdot\frac{1}{2}\right.$$

$$\left.\left(C_V^{\tilde{x}}-C_A^{\tilde{x}}\gamma^5\right)\bar{u}(P_1)\right)\times\mathrm{Tr}\left(u(P_4)\right.$$

$$\left.\gamma_{\mu}\frac{1}{2}\left(C_V^{\rho}-C_A^{\rho}\gamma^5\right)\bar{u}(P_2)\right)$$

with the trace formalism we can both switch matrices of a product without changing the result and by having a trace that is invariant under cyclic permutation. With this we can do the following below.

[wikipedia-artikel om Trace_(linear algebra)]

We can use the abovementioned properties to get:

$$\mathrm{Tr}\left(\left(\bar{u}(P_3)\gamma^{\mu}u(P_3)\right)\frac{1}{2}\left(C_V^{\tilde{x}}-C_A^{\tilde{x}}\gamma^5\right)\right)$$

$$\times\mathrm{Tr}\left(u(P_1)\gamma^{\nu}\bar{u}(P_1)\frac{1}{2}\left(C_V^{\tilde{x}}-C_A^{\tilde{x}}\gamma^5\right)\right)$$

$$\times\mathrm{Tr}\left(\bar{u}(P_4)\gamma_{\mu}u(P_4)\frac{1}{2}\left(C_V^{\rho}-C_V^{\rho}\gamma^5\right)\right)\times$$

$$\mathrm{Tr}\left(u(P_2)\,\gamma_\nu^\mu\,\bar{u}(P_2)\,\frac{1}{2}\left(c_V^\rho - c_A^\rho \gamma^5\right)\right)$$

With this we get and the Feynman slash notation, to become:

To calculate the unpolarized cross section, we average over initial spins and sum over final spins, with the factor 1/4.

$$\frac{1}{4}\sum_{spins}|M|^2 = \frac{-g_z^2}{q^2 - m_z^2}\, g^{\mu\nu} g_{\mu\nu}\,/$$

$$\mathrm{Tr}\left(\gamma^\mu (\not{P}_3 + m)\,\gamma^\nu (\not{P}_1 + m)\,\frac{1}{2}\left(c_V^x - c_A^x \gamma^5\right)\right)\mathrm{Tr}\left(\gamma_\mu (\not{P}_4 + m)\,\gamma_\nu (\not{P}_2 + m)\,\frac{1}{2}\left(c_V^f - c_V^f \gamma^5\right)\right)$$

We remember the contraction relations:

$$g^{\mu\nu} g_{\mu\nu} = 4$$

$$P_2^\mu P_1^\nu g_{\mu\nu} = P_1 \cdot P_2$$

$$P_2^\mu P_1^\nu P_{3\mu} P_{4\nu} = (P_2 \cdot P_3)(P_1 \cdot P_4)$$

We ignore the mass m and get the following relation:

$$= \left(P_3^{\mu} P_1^{\nu} - g^{\mu\nu}(P_3 \cdot P_1) + P_3^{\nu} P_1^{\mu}\right) \times$$
$$P_{4\mu} P_{2\nu} - g_{\mu\nu}(P_4 \cdot P_2) + P_{4\nu} P_{2\mu}\Big)$$
$$= \Big(P_3^{\mu} P_1^{\nu} P_{4\mu} P_{2\nu} - P_3^{\mu} P_1^{\nu} g_{\mu\nu}(P_4 \cdot P_2)$$
$$+ P_3^{\mu} P_1^{\nu} P_{4\nu} P_{2\mu} - g^{\mu\nu}(P_3 \cdot P_1) P_{4\mu} P_{2\nu} +$$
$$g^{\mu\nu} g_{\mu\nu}(P_3 \cdot P_1)(P_4 \cdot P_2) - g^{\mu\nu}(P_3 \cdot P_1)$$
$$(P_4 \cdot P_2) + P_3^{\nu} P_1^{\mu} P_{4\mu} P_{2\nu} - P_3^{\nu} P_1^{\mu} g_{\mu\nu}$$
$$(P_4 \cdot P_2) + P_3^{\nu} P_1^{\mu} P_{4\nu} P_{2\mu}\Big)$$
$$= (P_3 \cdot P_4)(P_1 \cdot P_2) - (P_1 \cdot P_3)(P_4 \cdot P_2) +$$
$$(P_3 \cdot P_2)(P_1 \cdot P_4) - (P_3 \cdot P_1)(P_4 \cdot P_2) +$$
$$4(P_3 \cdot P_1)(P_4 \cdot P_2) - (P_3 \cdot P_1)(P_4 \cdot P_2) +$$
$$(P_1 \cdot P_4)(P_3 \cdot P_2) - (P_1 \cdot P_3)(P_4 \cdot P_2) +$$
$$(P_1 \cdot P_2)(P_3 \cdot P_4)$$
$$= -4(P_1 \cdot P_3)(P_4 \cdot P_2) + 2(P_1 \cdot P_2)(P_3 \cdot P_4)$$
$$2(P_3 \cdot P_2)(P_1 \cdot P_4) + 4(P_3 \cdot P_1)(P_4 \cdot P_2)$$

$$= 2\,(P_1 \cdot P_2)(P_3 \cdot P_4) + 2\,(P_3 \cdot P_2)(P_1 \cdot P_4)$$

As can be seen in that particular relation, we have not included the two current terms we had earlier. This was done out of overview and space constraints.

So, we've used the cyclic property to take out the term $\gamma\left(\frac{1}{2}\left(C_V^{\tilde{x}} - C_A^{\tilde{x}}\gamma^5\right)\left(C_V^{g} - C_A^{g}\gamma^5\right)\right)$

We have the relations $(\gamma^5)^2 = 1$ and $\mathrm{Tr}(\gamma^5) = 0$. This gives us

$$\left(\frac{1}{2}\left(C_V^{\tilde{x}} C_V^{g} + C_A^{\tilde{x}} C_A^{g} (\gamma^5)^2 - 2\, C_V^{\tilde{x}} C_V^{g} C_A^{\tilde{x}} C_A^{g} (\gamma^5)^2\right)\right)^2$$

which becomes

$$\frac{1}{4}\left(\left(C_V^{\tilde{x}}\right)^2 + \left(C_A^{\tilde{x}}\right)^2 + \left(C_V^{g}\right)^2 + \left(C_A^{g}\right)^2 - \left[2\, C_V^{\tilde{x}} C_V^{g} C_A^{\tilde{x}} C_A^{g}\right]^2\right)$$

From Thomson we have the relations

$P_1 = (E, 0, 0, E)$, $P_2 = (E, 0, 0, -E)$

$P_3 = (E, E\sin\theta, 0, E\cos\theta)$, $P_4 = (E, -E\sin\theta, 0, -E\cos\theta)$

$p_1 \quad p_3 \qquad p_1 p_2 = p_3 p_4 \quad p_1 p_4 = p_2 p_3$

$p_2 \quad p_4 \qquad p_1 p_3 = p_2 p_4$

and this gives us in the COM frame (page 137–138 in Thomson)

$$p_1 \cdot p_2 = 2E^2, \quad p_1 \cdot p_3 = E^2(1-\cos\theta)$$
$$p_1 \cdot p_4 = E^2(1+\cos\theta)$$

so we then get the differential cross-section

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{-g_Z^2}{q^2 - m_Z^2} \frac{1}{4}\left(4E^4 + 2E^4(1+\cos\theta)^2\right)$$
$$\cdot\left(\left(C_V^{\tilde{\chi}}\right)^2 + \left(C_A^{\tilde{\chi}}\right)^2 + \left(C_V^{f}\right)^2 + \left(C_A^{f}\right)^2 - \left(2\, C_V^{\tilde{\chi}} C_A^{\tilde{\chi}} C_V^{f} C_A^{f}\right)^2\right)$$

we set $s = 4E^2 = q^2$ and get the following expression for the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{1}{4} \frac{-g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma^2}$$
$$\left[\frac{1}{4}\left(\left(\left(C_V^{\tilde{\chi}}\right)^2 + \left(C_A^{\tilde{\chi}}\right)^2 + \left(C_V^{f}\right)^2 + \left(C_A^{f}\right)^2\right)\cdot(1+\cos^2\theta) - \left(2\, C_V^{\tilde{\chi}} C_A^{\tilde{\chi}} C_V^{f} C_A^{f} \cos\theta\right)\right)\right]$$

We integrate over the solid angle $d\Omega = d\phi\, d(\cos\theta) = 2\pi\, d(\cos\theta)$ giving us

$$\int_{-1}^{+1} (1+\cos^2\theta)\, d(\cos\theta) = \int_{-1}^{+1} (1+x^2)\, dx = \frac{8}{3}$$

and $$\int_{-1}^{+1} \cos\theta \, d(\cos\theta) = 0.$$

where substitution rule has been used and ortogonality-principle respectively. So this gives us:

$$\sigma = \frac{1}{192\pi} \cdot \frac{-g_z^4}{(s-m_z^2)^2 + m_z^2\Gamma^2}$$

$$\left((C_V^{\tilde{\chi}})^2 + (C_A^{\tilde{\chi}})^2 + (C_V^{f})^2 + (C_A^{f})^2 \right)$$