

In this question we can write the force as  $F(t) = \frac{F_0 t}{T} (U(t) - U(t-T)) + F_0 U(t-T) = \frac{F_0 t}{T} U(t) - \frac{F_0(t-T)}{T} U(t-T)$ , again we will solve using DSLT.

1. Our equation is

$$m\ddot{x}(t) + \gamma\dot{x}(t) + kx(t) = F(t)$$

by DSLT:

$$mS^2 X(S) + \gamma S X(S) + kX(S) = \frac{F_0}{T} \left( \frac{1}{S^2} - \frac{e^{-TS}}{S^2} \right)$$

Let's define  $\alpha_1 = \frac{-\gamma + \sqrt{\gamma^2 - 4mk}}{2m}$  and  $\alpha_2 = \frac{-\gamma - \sqrt{\gamma^2 - 4mk}}{2m}$   
by algebraic manipulation:

$$X(S) = \frac{F_0}{T} \left[ \frac{1}{(S - \alpha_1)(S - \alpha_2)} \left( \frac{1}{S^2} - \frac{e^{-TS}}{S^2} \right) \right]$$

let's define:  $\hat{X}(S) = \frac{F_0}{T} \frac{1}{(S - \alpha_1)(S - \alpha_2)} \frac{1}{S^2}$   
by inverse DSLT:

$$\hat{x}(t) = \frac{F_0 t}{Tk} - \frac{F_0 \gamma}{Tk^2} + F_0 \gamma e^{-\frac{\gamma t}{2m}} \cdot y(t)$$

where

$$y(t) = \frac{\cosh\left(\frac{t\sqrt{\frac{\gamma^2}{4}-km}}{m}\right) - m \sinh\left(\frac{t\sqrt{\frac{\gamma^2}{4}-km}}{m}\right) \left(\frac{\gamma}{2m} - \frac{F_0\gamma^2 - F_0 km}{F_0\gamma m}\right)}{\sqrt{\frac{\gamma^2}{4}-km}}$$

and we get

$$x(t) = \hat{x}(t) U(t) - \hat{x}(t-T) U(t-T)$$