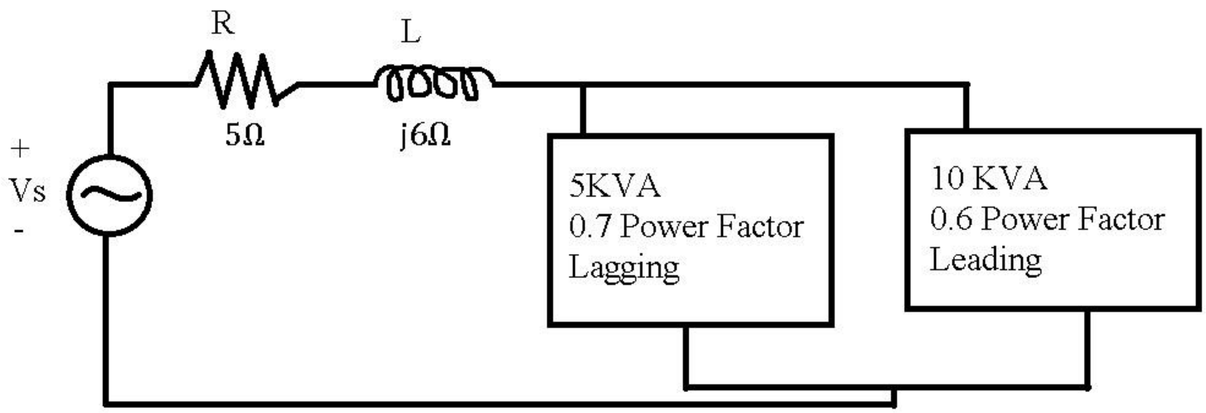


Load 1 is the 5 KVA load and Load 2 is the 10 KVA load. Loads 1 and 2 are operating at their rated voltages of 220 V rms.

a) Find Load 1's real and reactive power
b) Find Load 2's real and reactive power
c) Find the current leaving the source
d) Find the source's real and reactive power and completely draw its power triangle
e) Find the source's power factor
f) If a capacitive bank is placed on the source end, what reactive power should it provide so that the source operates at a power factor of 0.8 lagging.

(a)

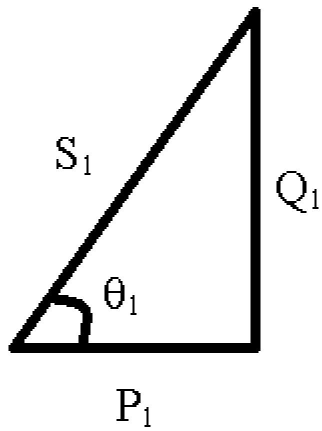
Let us call the voltage across the parallel loads V_1. We will take this node as the reference point with regards to the angle.

$$\overrightarrow{V_1} = 220\angle 0°$$

Let us call the complex power absorbed by Load 1 S_1

$$\left|\overrightarrow{S_1}\right| = 5\ KVA$$

Since the power factor for Load 1 is lagging, the reactive power will be positive, by convention. We also know from this information that Load 1 is inductive. The power triangle will look as follows:



We know that the power factor is equal to $\cos(\theta_1)$ so we can calculate the angle as follows:

$$\theta_1 = \cos^{-1}(0.7) = 45.572996°$$

With the value of θ_1, S_1, and the laws of trigonometry we find the values of Q_1 and P_1 as follows:

$$Q_1 = 5 \times 10^3 \times \sin 45.572996° = 3570.714214\ VAR \approx 3570\ VAR$$

$$P_1 = 5 \times 10^3 \times \cos 45.572996° = 3500\ W$$

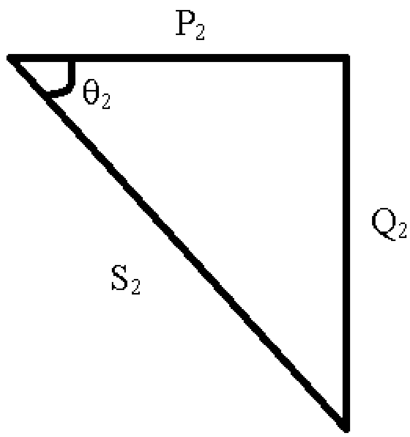
Where Q_1 is the reactive power and P_1 is the real power.

(b)

Let us call the complex power absorbed by Load 2 S_2.

$$\left|\overrightarrow{S_2}\right| = 10\ KVA$$

Since the power factor for Load 2 is leading, the reactive power will be negative, by convention. We also know from this information that Load 2 is capacitive. The power triangle will look as follows:



$$\theta_2 = \cos^{-1} 0.6 = 53.13010235°$$

$Q_2 = 10 \times 10\text{^}3 \times \sin 53.13010235° = -8000\ VAR$ (The value is negative because we are taking the leading power factor into account.)

$$P_2 = 10 \times 10^3 \times \cos 53.13010235° = 6000\ W$$

(c)

When the current leaving the source reaches the node that is the junction point between the two parallel loads, it divides itself between the two loads. If we calculate the current through each load and add them together, then the sum will equal the current leaving the source.

$$S_1 = V_1 {I_1}^* = 5 \times 10^3 \angle 45.572996°$$

$$\frac{S_1}{V_1} = \frac{V_1 I_1^*}{V_1} = I_1^* = \frac{5 \times 10^3 \angle 45.572996°}{220\angle 0°} = 22.72727273\angle 45.572996°$$

$$\therefore I_1 = 22.72727273\angle -45.572996°\ A$$

$$S_2 = V_1 {I_2}^* = 10 \times 10^3 \angle -53.13010235°$$

$$\frac{S_2}{V_1} = \frac{V_1 I_2^*}{V_1} = I_2^* = \frac{10 \times 10^3 \angle -53.13010235°}{220\angle 0°} = 45.45454545\angle -53.13010235°$$

$$\therefore I_2 = 45.45454545\angle 53.13010235°\ A$$

$$\begin{aligned} I = I_1 + I_2 &= 15.90909091 - j16.23051916 + 27.27272727 + j36.36363636 \\ &= 43.18181818 + j20.13311676 \end{aligned}$$

$$\vec{I} = 47.64464096\angle 24.99684394°\ A \approx 47.6\angle 25°\ A$$

(d)

Since we already have the current leaving the source, the next step is to find the voltage supplied by the source. We can find this by first finding the equivalent impedance of the entire circuit. We can then multiply the equivalent impedance by the current to get the voltage.

We begin by finding the equivalent impedance of the parallel loads. We already know that the voltage at the junction node is V_1 and we know the current is I. Using Ohm's Law we can find the impedance as follows:

$$\frac{V_1}{I} = Z_L = \frac{220\angle 0°}{47.64464096\angle 24.99684394°} = 4.617518268\angle -24.99684394°\ \Omega$$
$$= 4.18500025 - j1.951217022\ \Omega$$

We next add the equivalent load impedance to the resistor and inductor impedance since they are in series as follows:

$$Z_T = 5 + j6 + 4.18500025 - j1.951217022 = 9.18500025 + j4.048782978$$
$$= 10.03777232\angle 23.78806728°$$

Next, we multiply the current by the equivalent impedance to get the voltage supplied by the source as follows:

$$V_s = IZ_T = (47.64464096\angle 24.99684394°)(10.03777232\angle 23.78806728°)$$
$$= 478.2460582\angle 48.78491122°\ V$$

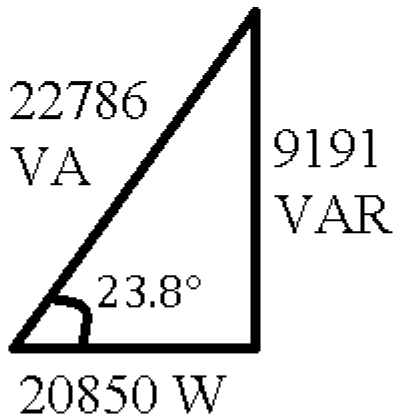
With the value of the voltage supplied by the source and the current leaving the source, we can find the power supplied by the source as follows:

$$S = V_s I^* = (478.2460582\angle 48.78491122°)(47.64464096\angle -24.99684394°)$$
$$= 22785.86173\angle 23.78806728° = 20850.05905 + j9190.785181 = P + jQ$$

Source real power $\approx 20850\ W$

Source reactive power $\approx 9191\ VAR$

We now have all the information necessary to draw the power triangle of the source as follows:



(e)

The power factor can be calculated by taking the cosine of the angle in the power triangle as follows:

$\cos 23.78806728 = 0.9150436925 \text{ Lagging.} \approx 0.915$ Lagging

We know the power factor is lagging because we compare the angles of the current and voltage and see that the current is lagging the voltage.

(f)

The angle that produces a power factor of 0.8 can be calculated as follows:

$$\cos^{-1} 0.8 = 36.86989765°$$

The reactive power needed in the power triangle to produce this angle would be as follows:

$$20850.05905 \tan 36.86989765° = 15637.54429\ VAR$$

The change in reactive power is as follows:

$$15637.54429 - 9190.785181 = 6446.759107$$

So, the capacitive bank would have to provide approximately 6447 VAR. Since this is a positive number, we are actually referring to an inductor, because a capacitor absorbs negative reactive power.