

$$\left\{ \begin{array}{l} 12 - 2\dot{Q}_1 - 3\dot{Q}_2 - \frac{Q_2}{C} = 0 \text{ (mesh 1)} \\ 12 - 2\dot{Q}_1 - 5\dot{Q}_3 = 0 \text{ (mesh 3)} \\ \dot{Q}_1 = \dot{Q}_2 + \dot{Q}_3 \\ +3\dot{Q}_2 + \frac{Q_2}{C} - 5\dot{Q}_3 = 0 \text{ (mesh 2)} \end{array} \right.$$

By the equation 4,

$$\dot{Q}_3 = \frac{3}{5}\dot{Q}_2 + \frac{Q_2}{5C}$$

By the equation 3,

$$\dot{Q}_3 = \dot{Q}_1 - \dot{Q}_2$$

So

$$\dot{Q}_1 - \dot{Q}_2 = \frac{3}{5}\dot{Q}_2 + \frac{Q_2}{5C}$$

But by the equation 1,

$$\dot{Q}_1 = 6 - \frac{3}{2}\dot{Q}_2 - \frac{Q_2}{2C}$$

Finally, we get an equation with an unknown:

It is a system of 3 differential equations. It is not a system of algebraic equations (which we solve by Kramer, scaling, substitution); There are standardized methods for solving ODE systems... (which use concepts of eigenvalues and eigenvectors...), so it's not a simple exercise. Some cases are even simple, because they can be solved by substitution. That's what we're going to try here. I hope it will work, but if it doesn't, that's fine and we will have to resort to other methods... (even Laplace's transformation may help...) (I wrote 4 equations between the keys because I used all the meshes, but one of them is redundant and we only need 3 of them for the 3 unknowns). Didactically in life we start by learning systems with two differential equations and 2 unknowns, but here 3 appeared...

$$6 - \frac{3}{2} \dot{Q}_2 - \frac{Q_2}{2C} - \dot{Q}_2 = \frac{3}{5} \dot{Q}_2 + \frac{Q_2}{5C}$$

$$3,1 \dot{Q}_2 + 0,7 \frac{Q_2}{C} = 6$$

It is a linear first-order differential equation, with constant and inhomogeneous coefficients. The initial condition (not given in the statement, but we assume a capacitor fully discharged initially) is $Q_2(t=0) = 0$. Therefore, solving on the site (although it is possible to solve analytically using integral factor or variable separation)

<https://www.wolframalpha.com/input?i=3.1y%27+%2B0.7y%2FC+%3D+6%2C+y%280%29%3D0>

I got as a solution:

$$Q_2(t) = C(8,857143 - 8,857143e^{-0,225806t/C})$$

The value of the capacitance C was given in the statement as $C = 200 \text{ pF}$, so rounding up, we are left with

$$Q_2(t) \approx 1,8 \cdot 10^{-9}(1 - e^{-10^9 t})$$

Checking: for $t=0$, we get that the charge is zero. And for $t \rightarrow \infty$, the charge is constant (so the current, which is its derivative, zeroes). The current is $I_2 = \dot{Q}_2 = 1,8e^{-10^9 t}$, which depends on the time.

Using that

$$\dot{Q}_1 = 6 - \frac{3}{2} \dot{Q}_2 - \frac{Q_2}{2C}$$

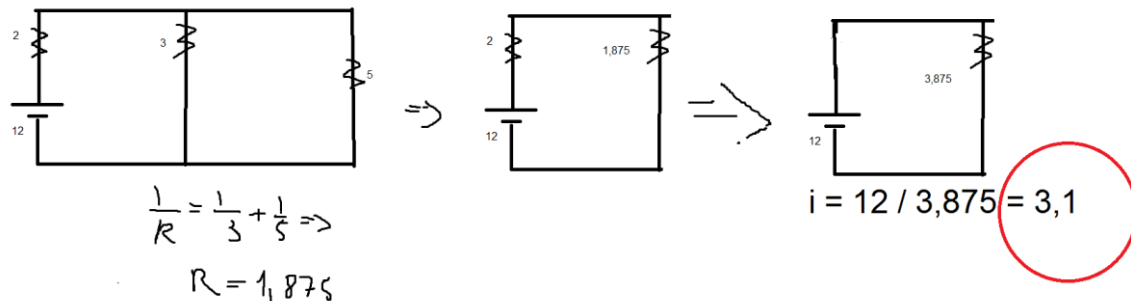
We get the current $I_1 = \dot{Q}_1$:

$$I_1 = \dot{Q}_1 = 6 - \frac{3}{2} 1,8e^{-10^9 t} - \frac{1,8 \cdot 10^{-9}(1 - e^{-10^9 t})}{2 \cdot 200 \cdot 10^{-12}} = 1,5 + 1,8e^{-10^9 t}$$

By the equation above, for $t=0$, the current will be $I_1 = 1,5 + 1,8 \cdot 1 = 3,3$

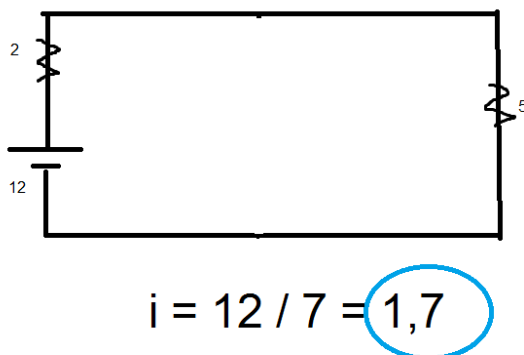
And for $t \rightarrow \infty$, the current is 1.5 (because the exponential will zero) (pay attention to the many roundings we did above...)

The initial current can be obtained by the initial figure, without having to solve the EDO system and ignoring the capacitor, since it was discharged:



The difference between 3.3 and 3.1 was probably that bunch of rounding we did above to get to 3.3. Please check it out....

The final current (charged capacitor) can be obtained by the initial figure, without having to solve the EDO system and ignoring the capacitor and resistor, since no current passes "in the middle wire":



Again, the difference was probably the rounding we did to get to the 1,5

The current I_3 will be

$$I_3 = I_1 - I_2 = 1,5 + 1,8e^{-10^9 t} - 1,8e^{-10^9 t} = 1,5$$

That doesn't depend on the time

Could it have been possible to have predicted this without first solving the equations?