



Using KVL...

$$-V_C(t) + V_R(t) = 0$$

$$V_R(t) = i_R(t) R \dots$$

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because it is a series circuit, $i_R(t) = i_C(t) = C \frac{dV(t)}{dt} \dots$

$$V_C(t) = \left(C \frac{dV(t)}{dt} \right) R = RC \frac{dV(t)}{dt}$$

rearranging...

$$\frac{dV_C(t)}{dt} - \frac{V_C(t)}{RC} = 0$$

Using an integrating factor...

$$u(t) = e^{\int -1/RC dt} = e^{-t/RC + K} = e^K e^{-t/RC} = K e^{-t/RC}$$

$$K e^{-t/RC} \left(\frac{dN_c(t)}{dt} - \frac{N_c(t)}{RC} \right) = 0$$

left hand side is equivalent to...

$$\frac{d}{dt} \left(K e^{-t/RC} N_c(t) \right)$$

So...

$$\frac{d}{dt} \left(K e^{-t/RC} N_c(t) \right) = 0$$

integrating to eliminate derivative...

$$\int \frac{d}{dt} \left(K e^{-t/RC} N_c(t) \right) dt = \int 0 dt$$

$$K e^{-t/RC} N_c(t) + K_2 = K_3$$

Solving for $N_c(t)$ and combining constants where appropriate...

$$N_c(t) = K e^{t/RC}$$

using initial values to solve for K ...

$$N_c(0) = N_0 = K e^{(0)} = K$$

so...

$$N_c(t) = N_0 e^{t/RC} \leftarrow \text{should be } e^{(-t/RC)}$$

Notice if I switch the polarity of either C or R the equation becomes correct...