Standing Waves on a Sonometer

Goal: To study the properties of standing waves on a string and compare a theoretical calculation of the velocity of the wave on the string with an experimental value of the velocity.

Equipment List:

Pasco Sonometer base
2 bridges
 2 wires
1 kg mass
1 driver and detector coil with BNC connectors
JDR dual trace oscilloscope (or Fluke scope)
Pasco Digital Function generator
1BNC Tee connector
2 BNC to BNC connectors
small bubble level
digital mass balance
meter stick (To measure the length of the string)
loop of cord to support 1 kg mass
vernier caliper

Background: When two traveling waves with the same amplitue, wave number and frequency travel different directions on a string fixed at both ends, the principle of superposition of the two wave functions $y_1(x,t) = Asin(kx - wt)$ and $y_2(x,t) = Asin(kx + wt)$ yields:

$$y(x,t) = 2A\ cos(wt)sin(kx) \qquad \text{(Eq. 1)}$$

<u>Pre-lab exercise:</u> Derive this expression.

Equation 1 describes a wave fixed in space with an amplitude varying in time according to *2Acos(wt)*. This is a standing wave. This wave will have positions of zero amplitude, called nodes, and positions of maximum amplitude, called antinodes. The velocity of the wave on this string is given by:

$$v = (T/\mu)^{\frac{1}{2}} \qquad\qquad \text{(Eq. 2)}$$

where T is the tension and μ is the mass per unit length. The velocity of the wave on the string can also be expressed in terms of the wavelength and frequency:

$$v = f*lambda \qquad\qquad \text{(Eq. 3)}$$

Since, in the case of the string fixed at both ends

$$y(0, t)\ = y(L, t) = 0 \qquad\qquad \text{(Eq. 4)}$$

or...

$$sin(k*0) = sin(k*L) = 0 = sin(n*pi) \qquad \text{for n} = 1, 2, 3... \ \ \text{(Eq. 5)}$$

Recall that the wave number k is:

$$k = 2*pi/lambda$$

Thus, combining the definition of k with equation 5
$$kL = (2*pi/lambda)L = n*pi \qquad \text{(Eq. 6)}$$
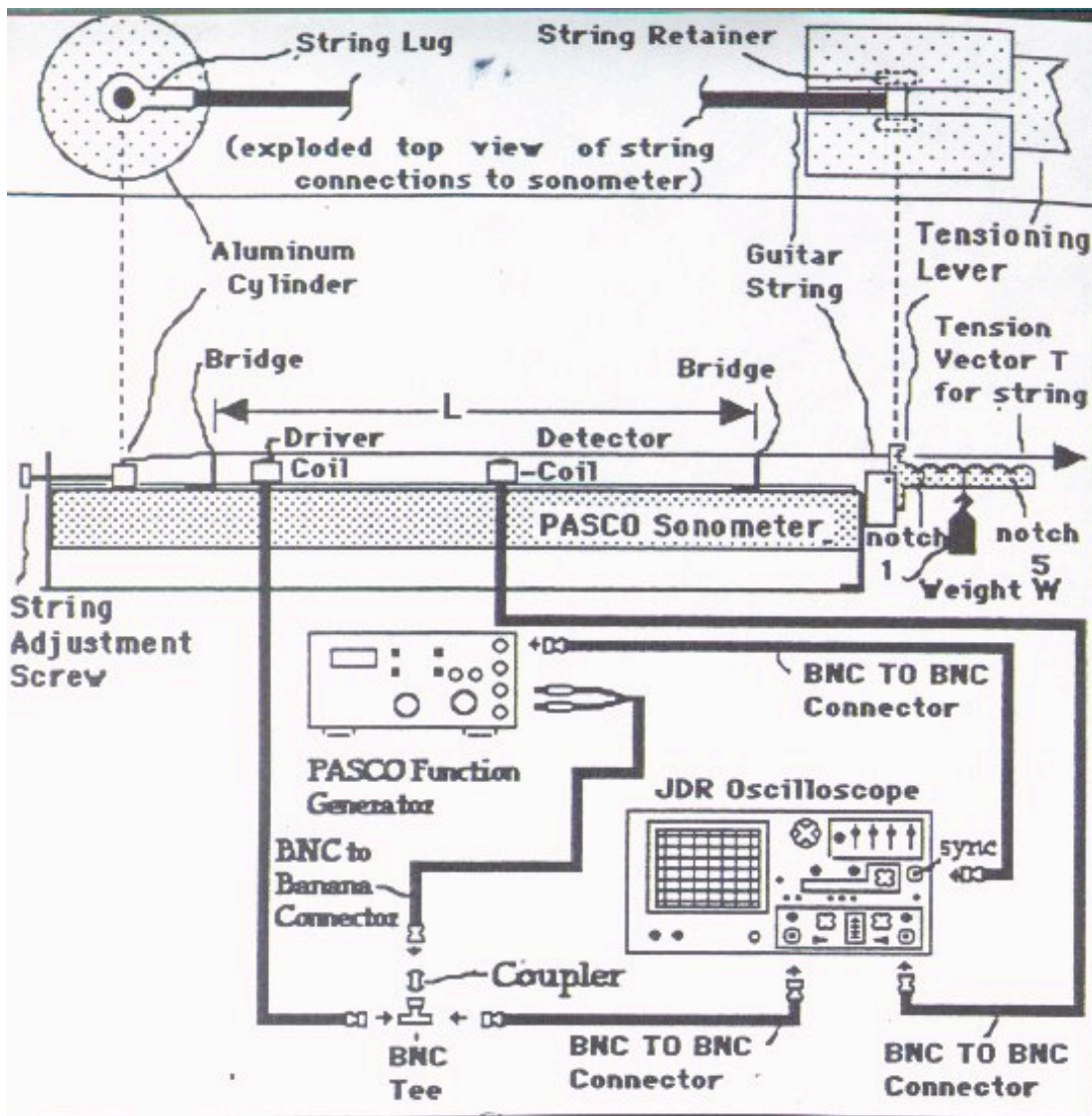 or...
$$L = n*lambda/2 \qquad \text{(Eq. 7)}$$
This gives you an expression for the wavelength in terms of the length of the string, *L*, and, *n*, the mode of vibration. For the fundamental mode, *n = 1,* and the wavelength, *lambda,* is *2L.*

Combining equations 2, 3, and 7 to obtain an expression for the frequency:
$$f = v/lambda = (T/\mu)^{1/2}/(2L/n) \; = n(T/\mu)^{1/2}/2L \quad (8)$$

Procedure:

(exploded top view of string connections to sonometer)

- Set up the Sonometer as shown.
- Select a guitar string from the bag. Measure the total mass of the string. The mass of the connecting lugs on the end is 0.89 x 10^{-3}kg and should be subtracted from the measured mass. Measure the

length with a meter stick. Compute the mass/length μ. Compare with the values given by Pasco:

- 0.010 in. diameter, $\mu = 3.9 \times 10^{-4}$ kg/m
- 0.014 in. diameter, $\mu = 7.8 \times 10^{-4}$ kg/m
- 0.017 in. diameter, $\mu = 1.12 \times 10^{-3}$ kg/m
- 0.021 in. diameter, $\mu = 1.50 \times 10^{-3}$ kg/m
- 0.023 in. diameter, $\mu = 1.84 \times 10^{-3}$ kg/m

- Place the string on the sonometer base and put the bridges about 60 cm apart.
  - Hang a mass of approximately 1 kg from the tensioning lever.
    - Note that the tension T in the string is given by the notch# times the weight: $T = (notch\#)(W)$.
    - Adjust the string adjustment knob so that the tensioning lever is horizontal.
  - Position the driver coil approximately *5 cm* from one of the bridges and position the detector near the center of the wire.
  - Record the length, tension (mg), and linear density of the string.
- Connect the driver coil and detector coil to the Pasco generator and oscilloscope using a BNC Tee, coupler (if necessary) and BNC connectors as shown.
- Set the pasco digital function gernerator as follows:
  - sine wave output
  - amplitude: mid range
  - frequency selector: 25 Hz
- Set the oscilloscope controls initially as follows
  - Mode: dual
  - ChA vertical gain: 2 to 5 volts/cm
  - ChB vertical gain: 20 mvolts/cm
  - trigger: INT ChA (only for JDR scopes)
- Slowly increase the frequency of the signal to the driver coil, starting at approximately 25 Hz. Listen for an increase in the volume of the sound from the sonometer and/or an increase in the size of the detector signal on the oscilloscope window. Frequencies that result in maximum string vibration are resonant frequencies. Determine the lowest frequency at which resonance occurs. This is resonance in the first, or fundamental, mode. Measure and record this frequency both from the function generator and from the oscilloscope trace.
  - IMPORTANT!!! NOTE: The driving frequency of the signal generator may not be the frequency at which the wire is vibrating. By observing the trace on the oscilloscope, you can determine if the two frequencies are the same, or if the vibrating frequency is a multiple of the driving frequency. The frequency observed on the wire is often **twice** the driver frequency because the driver electromagnet exerts a force on the wire twice during each cycle. The frequency found from the oscilloscope **is** the frequency of vibration of the string.   RECORD the frequency from the oscilloscope.
- Continue increasing the frequency to find successive resonant frequencies (approximately five). With the fundamental, there are nodes only at the bridges and the antinode is in the center. When you have the string vibrating in a higher mode, find the nodes and antinodes as follows:
  - Start with the detector as close as you can get it to one of the bridges. Watch the oscilloscope as you slide the detector slowly along the string.
  - Locate and record the locations of each node and antinode.
- Record the resonant frequency for each mode and the locations of nodes and antinodes.

Analysis:

From your results, determine and record the wavelength of each resonance pattern you discovered.
ADJACENT NODES ARE ONE HALF WAVELENGTH APART!
Determine the velocity for each of your measurements. You should have five values for the velocity of the wave on the string, and all should represent the same value given by:

$$v = f_n*(lambda)_n \qquad n = 1, 2, 3...$$

    Where

$$(lambda)_n = 2L/n \qquad n = 1, 2, 3... \text{ is found by examining the position of the}$$

nodes.

        and $\qquad f_n = nf_1$
        is found from the oscilloscope.

For each string length, calculate the average velocity, the standard deviation and the standard error.

Compare this value for the velocity with that obtained from the tension and mass per unit length. Using error propagation techniques, compute the velocity of the wave on the string and the uncertainty.

$$v = (T/\mu)^{1/2}$$

Conclusion:
  Compare your velocity calculated from frequency and wavelength considerations with the velocity based on the tension and mass per unit length.
Do your results agree within uncertainty? Why or why not?

An AVI movie of the setup