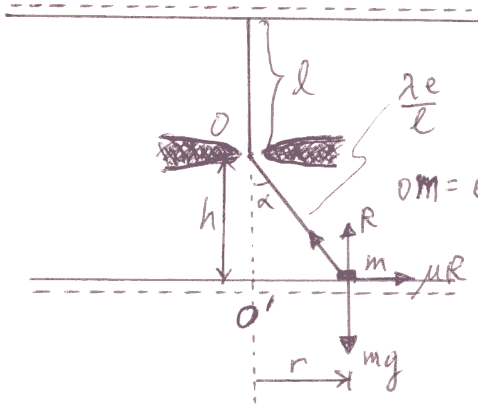
Solutions to Problem Set #2 of PHYS1414 (1st. semester, 2009-2010)

1.



$$\begin{cases} \dfrac{\lambda e}{l}\cos\alpha + R = mg \\ \dfrac{\lambda e}{l}\sin\alpha \leq \mu_0 R \end{cases}$$

$$\Rightarrow \quad \frac{\lambda e}{l}\sin\alpha \leq \mu_0\left[mg - \frac{\lambda e}{l}\cos\alpha\right]$$

$$\Rightarrow \quad \frac{\lambda r}{l} \leq \mu_0\left[mg - \frac{\lambda h}{l}\right] \qquad \begin{cases} e\sin\alpha = r \\ e\cos\alpha = h \end{cases}$$

$$r \leq \mu_0\left[\frac{mgl}{\lambda} - h\right]$$

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2. (a)

$$F - kv^2 = m\ddot{x}$$

When $v = u$ $\quad F - ku^2 = 0$

$$\Rightarrow \quad F = ku^2$$

When the engines are reversed, the motion of the ship is governed by $\quad -F - kv^2 = m\ddot{x}$

$$\Rightarrow \quad -ku^2 - kv^2 = m\ddot{x}$$

$$\Rightarrow \quad -k(u^2 + v^2) = m\frac{dv}{dt}$$

$$\Rightarrow \quad -\frac{k}{m}\int_0^t dt = \int_u^0 \frac{dv}{u^2 + v^2}$$

$$\Rightarrow \quad -\frac{k}{m}t = \frac{1}{u}\int_{\frac{\pi}{4}}^0 d\theta \qquad (v = u\tan\theta)$$

$$\Rightarrow \quad -\frac{k}{m}t = -\frac{1}{u}\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad t = \frac{m\pi}{4ku}$$

$$\Rightarrow \quad t = \frac{m\pi u}{4F} \qquad (F = ku^2)$$

\#

(b) $-F-kv^2 = m\ddot{x}$

$-ku^2-kv^2 = m\ddot{x}$

$-k(u^2+v^2) = mv\dfrac{dv}{dx}$

$$-k\int_0^x dx = m\int_u^0 \frac{v\,dv}{u^2+v^2}$$

$$-kx = \frac{m}{2}\int_{v=u}^{0} \frac{dv^2}{u^2+v^2}$$

$$-kx = \frac{m}{2}\ln(u^2+v^2)\Big|_{v=u}^{0}$$

$$-kx = \frac{m}{2}\left[\ln u^2 - \ln 2u^2\right]$$

$$-kx = -\frac{m}{2}\ln 2$$

$$x = \frac{m\ln 2}{2k}$$

$$x = \frac{mu^2\ln 2}{2F} \qquad (F = ku^2)$$

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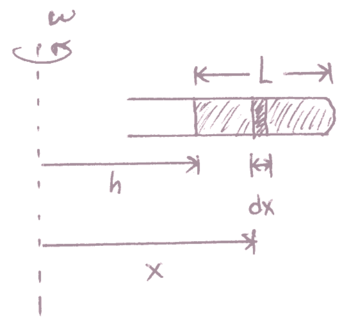
3.

The force on the small volume of liquid



$$S\,\underbrace{dp}_{\text{pressure difference}} = \underbrace{\rho\, dx\, S}_{\text{mass}}\, w^2 x$$

Cross sectional area

$$\Rightarrow \quad dp = \rho w^2 x\, dx$$

$$\Rightarrow \quad \int_{p_0}^{p} dp = \rho w^2 \int_{h}^{L+h} x\, dx$$

$$p - p_0 = (\rho w^2) \frac{x^2}{2} \Big|_{h}^{L+h}$$

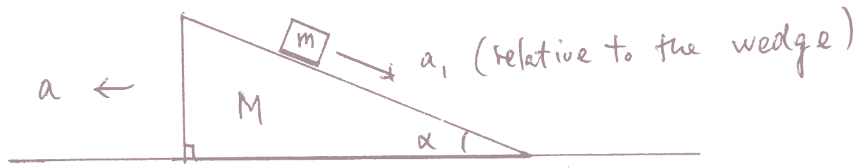
$$p - p_0 = \frac{\rho w^2}{2}\left[(L+h)^2 - h^2 \right]$$

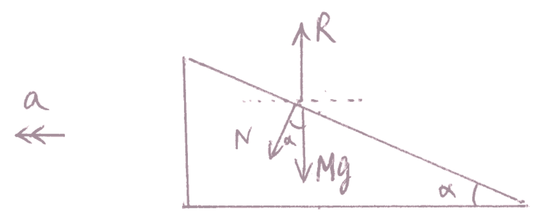
$$p = p_0 + \frac{\rho w^2}{2} L (L + 2h) \quad \divideontimes$$

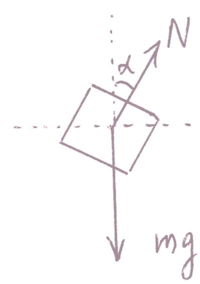
4.

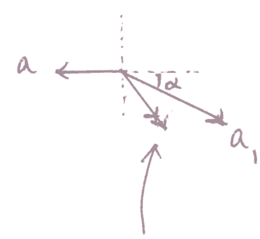
Consider the wedge M :

$$\begin{cases} \parallel \text{ to horiz}: & N\sin\alpha = Ma & (*) \\ \perp \text{ to horiz}: & N\cos\alpha + Mg = R & (**) \end{cases}$$



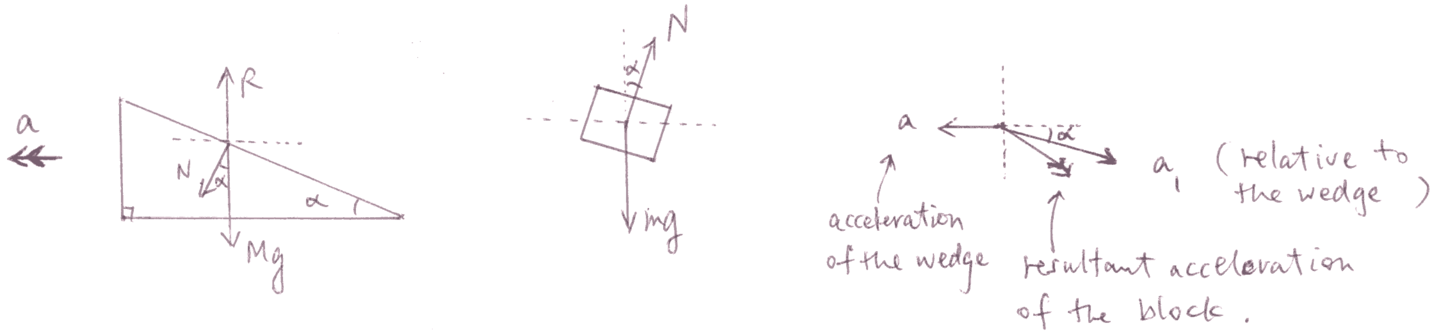






resultant acceleration of the block

Method I : Relative to an inertial observer



(a) Consider the block m:

$$\begin{cases} \parallel \text{ to the horiz: } & N\sin\alpha = m(a_1\cos\alpha - a) \quad \text{—— } ① \\ \perp \text{ to the horiz: } & mg - N\cos\alpha = ma_1\sin\alpha \quad \text{—— } ② \end{cases}$$

(b) Recall that from (*) $N\sin\alpha = Ma$ (*)

$$\begin{aligned} ①, (*) &\Rightarrow \quad m(a_1\cos\alpha - a) = Ma \quad \text{—— } ③ \\ ②, (*) &\Rightarrow \quad mg\sin\alpha - Ma\cos\alpha = ma_1\sin^2\alpha \quad \text{—— } ④ \end{aligned}$$

$$③ \Rightarrow \quad a_1 = \frac{(M+m)a}{m\cos\alpha} \quad \text{—— } ⑤$$

$$④, ⑤ \Rightarrow \quad mg\sin\alpha = \left[\frac{(M+m)}{\cos\alpha}\sin^2\alpha + M\cos\alpha\right]a$$

$$\Rightarrow \quad a = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$$

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Hence, we have $a_1 = \dfrac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$

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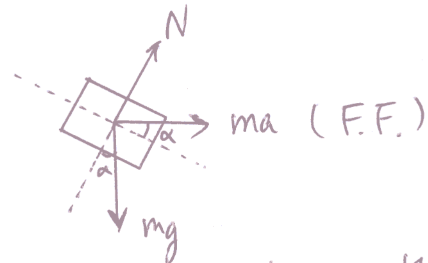
Method II : Relative to the wedge M

(As observed by an inertial observer)



(For observer sitting on the wedge)

(a) Consider the block m :

$$\begin{cases} \parallel \text{ to the incline} : \underbrace{ma\cos\alpha}_{\text{fictitious force}} + mg\sin\alpha = ma_1 & \text{—— ⑥} \\ \perp \text{ to the incline} : N - mg\cos\alpha + \underbrace{ma\sin\alpha}_{\text{fictitious force}} = 0 & \text{—— ⑦} \end{cases}$$

(b) Recall that from (*) $\quad N\sin\alpha = Ma \quad$ (*)

$$\begin{aligned} ⑥ &\Rightarrow \begin{cases} a\cos\alpha + g\sin\alpha = a_1 & \text{—— ⑧} \end{cases} \\ ⑦,(*) &\Rightarrow \begin{cases} Ma - mg\cos\alpha\sin\alpha + ma\sin^2\alpha = 0 & \text{—— ⑨} \end{cases} \end{aligned}$$

$$⑨ \Rightarrow \quad a = \frac{mg\cos\alpha\sin\alpha}{M + m\sin^2\alpha}$$

*

Hence, we have $\quad a_1 = \dfrac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$

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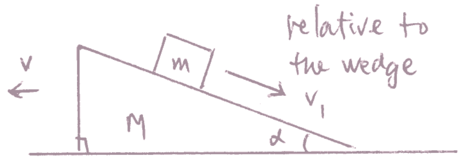
Conservation of linear momentum along the horizontal:

$$-Mv + m(v_1\cos\alpha - v) = 0$$



$$v = \frac{m v_1 \cos\alpha}{M+m}$$

$$\boxed{\frac{dv}{dt} = a} \qquad \boxed{\frac{dv_1}{dt} = a_1}$$

$$\Rightarrow \quad a = \frac{m a_1 \cos\alpha}{M+m} \quad \text{——} \quad (***)$$

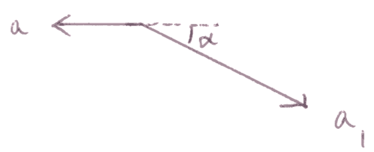
※

Plug a, a_1 into $(***)$, the equality holds.

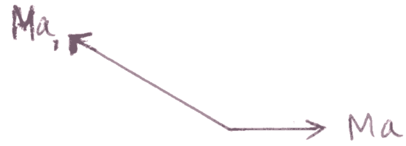
Recall
$$\begin{cases} a = \dfrac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha} \\[2ex] a_1 = \dfrac{(M+m)g\sin\alpha}{M+m\sin^2\alpha} \end{cases}$$

Motion of the wedge relative to the block

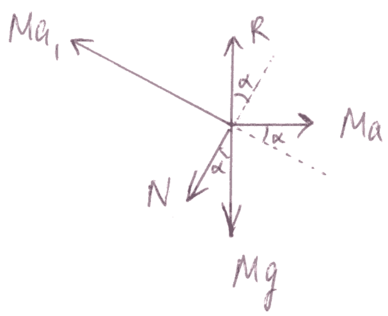
Acceleration of the block
as observed from an inertial frame.



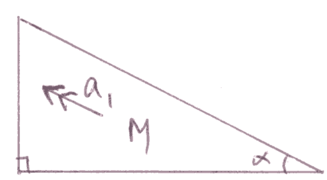
F.F. acting on the wedge



Total forces to account the
equation of motion of the wedge



Acceleration of the wedge
as observed by the block.



// to the incline:

$$Mg\sin\alpha \underbrace{- Ma_1 + Ma\cos\alpha}_{\text{fictitious force}} - R\sin\alpha = -Ma_1$$

$$\Rightarrow \quad Mg\sin\alpha + Ma\cos\alpha - R\sin\alpha = 0 \qquad \text{—— (10)}$$

⊥ to the incline:

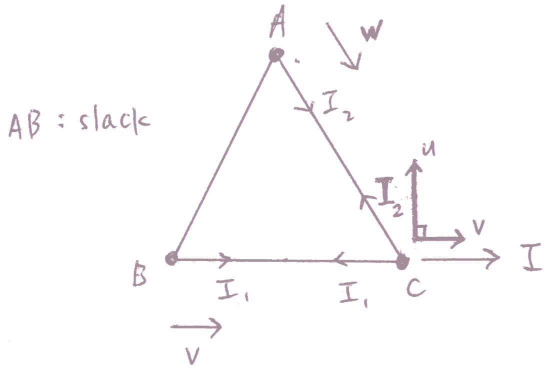
$$N + Mg\cos\alpha \underbrace{- Ma\sin\alpha}_{\text{fictitious force}} - R\cos\alpha = 0 \qquad \text{—— (11)}$$

To check the correctness of ⑩ & ⑪, let's plug in the following items.

$$\begin{cases} a = \dfrac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha} & N = \dfrac{mMg\cos\alpha}{M+m\sin^2\alpha} \\[2ex] a_1 = \dfrac{(M+m)g\sin\alpha}{M+m\sin^2\alpha} & R = \dfrac{Mg(M+m)}{M+m\sin^2\alpha} \end{cases}$$

6.

$$\begin{cases} I - I_1 - I_2 \cos 60^\circ = mv \\ I_2 \sin 60^\circ = mu \\ I_1 = mv \\ I_2 = mw \\ v\cos 60^\circ - u\cos 30^\circ = w \end{cases}$$



$$\Rightarrow \begin{cases} I - I_1 - \frac{I_2}{2} = mv & \text{— (1)} \\ \frac{\sqrt{3}}{2} I_2 = mu & \text{— (2)} \\ I_1 = mv & \text{— (3)} \\ I_2 = mw & \text{— (4)} \\ \frac{v}{2} - \frac{\sqrt{3}}{2} u = w & \text{— (5)} \end{cases}$$

(1), (3) and (4) $\Rightarrow$ $I - mv - \frac{mw}{2} = mv$

$\Rightarrow$ $I - 2mv - \frac{mw}{2} = 0$ — (6)

(2), (4) $\Rightarrow$ $\frac{\sqrt{3}}{2} w = u$ — (7)

Subst. it into (5) $\Rightarrow$ $\frac{v}{2} - \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2} w = w$

$\Rightarrow$ $\frac{v}{2} = \frac{7}{4} w$

$\Rightarrow$ $v = \frac{7}{2} w$ — (8)

⑥, ⑧ $\Rightarrow$ $$I - 2m\left(\frac{7}{2}w\right) - \frac{mw}{2} = 0$$

$$\Rightarrow \quad I = \frac{15mw}{2}$$

$$\Rightarrow \quad w = \frac{2I}{15m}$$ #

From ⑧ $$v = \frac{7I}{15m}$$ #

From ⑦ and the result of w, we have

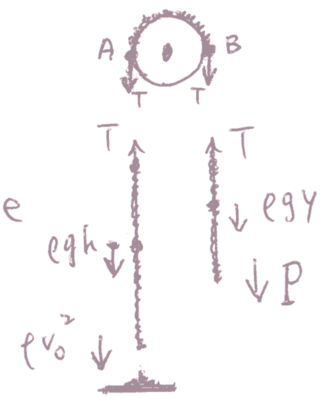
$$u = \frac{\sqrt{3}\,I}{15m}$$ #

$$V_B : V_A = v : w = \frac{7I}{15m} : \frac{2I}{15m} = 7 : 2$$ #

$$V_C = \sqrt{u^2 + v^2} = \frac{I}{m}\sqrt{\left(\frac{7}{15}\right)^2 + \frac{3}{(15)^2}} = \frac{2I}{m}\sqrt{\frac{13}{15}}$$ #

7.

Since the chain moves with uniform speed.



The tensions at pts. A and B are the same

$$\begin{cases} P + y\rho g - T = 0 \\ h\rho g + \underbrace{\rho v_0^2} - T = 0 \end{cases}$$

force pointing down : $\rho \dfrac{dy}{dt}(v_0) = \rho v_0^2$

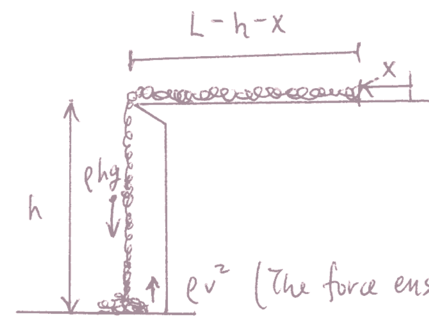
$$P + y\rho g - h\rho g - \rho v_0^2 = 0$$

$$P = \rho\left[hg - yg + v_0^2 \right]$$

#

8.

(a) $$\underbrace{-\rho v^2 + \rho h g}_{\text{net force on the chain (whole)}} = \frac{d}{dt}\Big[\overbrace{\rho(L-x)}^{\text{mass of moving chain}} v\Big]$$



(The force ensures the speed change eg. $v \to 0$)

$$\cancel{-v^2} + hg = (L-x)a - \cancel{v^2}$$

$$\begin{cases} \dot{x} = v \\ \ddot{x} = a \end{cases}$$

$$gh = (L-x)\ddot{x}$$

$$gh = (L-x)\frac{d\dot{x}}{dx}\dot{x}$$

$$\int_0^{L-h} \frac{gh}{L-x} = \int_0^{v_1} \dot{x}\, d\dot{x}$$

$$-gh\ln(L-x)\Big|_0^{L-h} = \frac{\dot{x}^2}{2}\Big|_0^{v_1}$$

$$-gh\ln\left(\frac{h}{L}\right) = \frac{v_1^2}{2}$$

$$v_1^2 = 2gh\ln\left(\frac{L}{h}\right)$$ #

(b) $$v_2^2 = v_1^2 + 2gh$$

* The end of chain (pt. A) moves down under gravity.

$$v_2^2 = 2gh\ln\left(\frac{L}{h}\right) + 2gh$$

$$v_2^2 = 2gh\left[1 + \ln\left(\frac{L}{h}\right)\right]$$

#