Definition 1 *Let X and Y be two normed spaces. We say that F is $\varepsilon{-}\delta{-}$u.s.c. if to every $x_0 \in X$ and $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$\|x - x_0\| < \delta \Longrightarrow F(x) \subseteq B_Y(F(x_0), \varepsilon).$$

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Where,

$B_Y(F(.), \delta) = \{y \in Y : d(y, F(.)) < \delta\}$*is called the ball in Y of radius a round the subset $F(.)$.*

Proposition 2 *Let X and Y be two normed spaces, F is a set-valued map from X to Y. Then we have:*

1. *If F is u.s.c., then F is $\varepsilon - \delta-$u.s.c. The converse is true if F has compact values.*

2. *If F is $\varepsilon - \delta-$l.s.c., then F is l.s.c. The converse is true if F has compact values.*

Example 3 *To show that the converse of the assertion **1**. in the preceding proposition is not true if the values of F are not compact consider the following example. Let $F : \mathbb{R} \longrightarrow P(\mathbb{R}^2)$ be defined by $F(\lambda) = \{(x, y) : x = \lambda,\ y \in \mathbb{R}\}$. Clearly F is $\varepsilon - \delta-$u.s.c. at each point in $\mathbb{R}$ (by taking $\varepsilon = \delta$). Now, consider the open subset $N = \{(x, y) : |xy| < 1\}$. which contains $F(0) = \{(0, y) : y \in \mathbb{R}\}$ We remark that for any $x \neq 0$, $F(x)$ is not contained in N. Thus, F is not u.s.c. at 0.*

Example 4 *To show that the converse of the assertion **2**. in the preceding proposition is not true if the values of F are not compact consider the following example. Let $F : [0, 1] \longrightarrow P(\mathbb{R}^2_+)$ be defined by $F(x) = \{(t, xt) : t \in \mathbb{R}_+\}$. Obviously F is l.s.c. at each point in $[0, 1]$. But F is not $\varepsilon - \delta-$l.s.c. Since for any $x_0, x \in [0, 1]$ and $x \neq x_0$, sup $\{d(y, F(x)) : y \in F(x_0)\} = \infty$ which implies that $F(x_0) \nsubseteq B(F(x), \varepsilon)$, for any $0 < \varepsilon < \infty$.*

The questions are:
1/ Show that F is $\varepsilon - \delta-$u.s.c. in Example (3) ?
2/ Show that F is l.s.c. in Example (4)?